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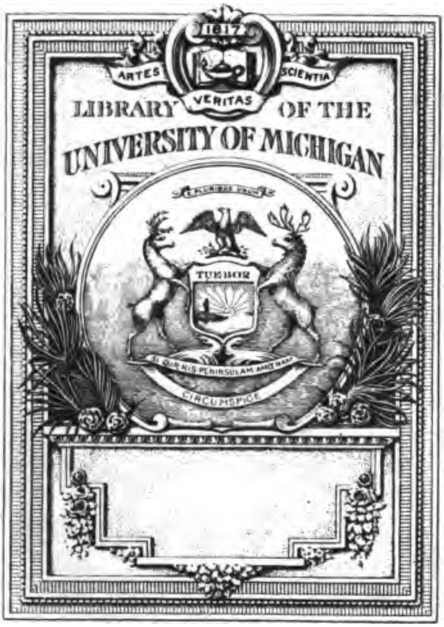
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Rule for finding the noumber of sq inches in a Circle Multeply the number of inches in the diameter by Its self divide the prouct by 14 and Multeply the quotient by 11 the result will give the sq inches

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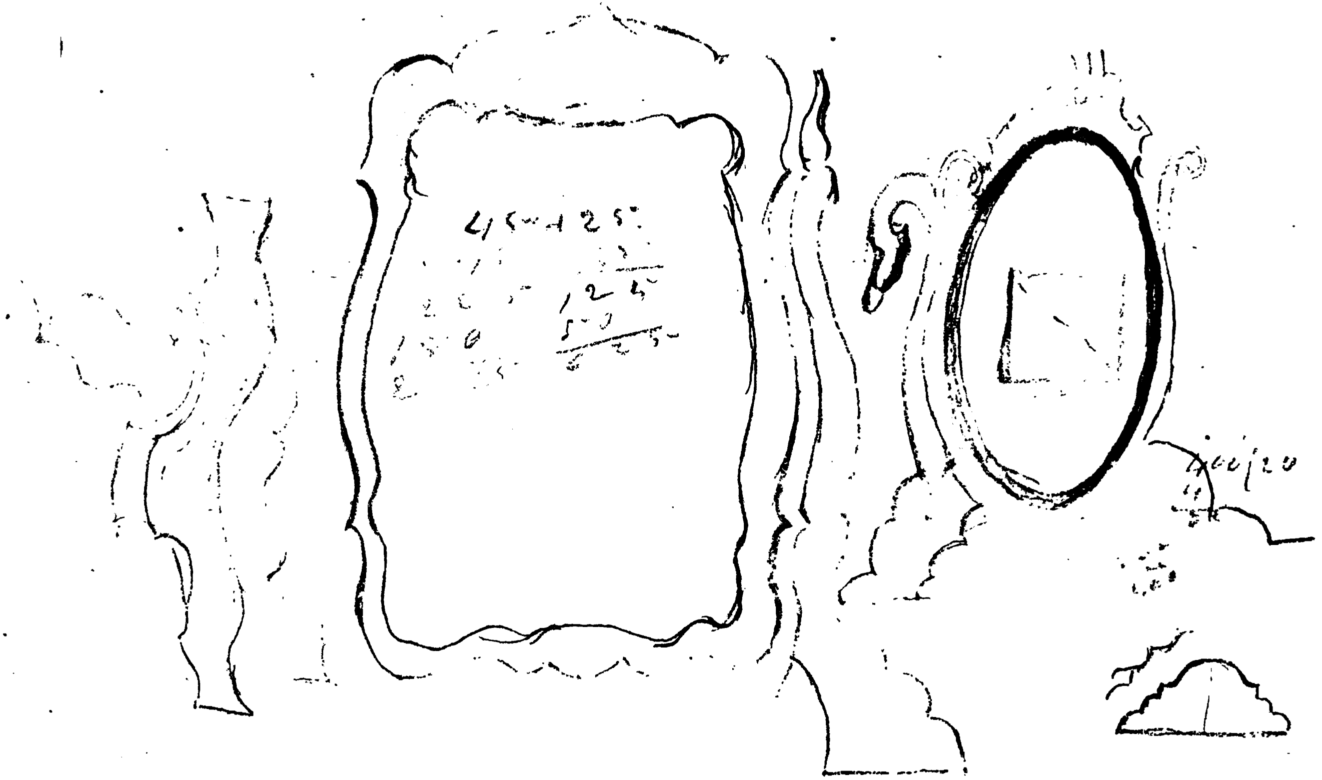
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A

NEW SYSTEM

OF

ARITHMETIC,

ON THE

CANCELLING PLAN:

EMBRACING THE

RULES OF THREE, SINGLE AND DOUBLE, DIRECT AND INVERSE;
BARTER; LOSS AND GAIN; REDUCTION, MULTIPLICATION
AND DIVISON OF FRACTIONS; EXCHANGE OF
CURRENCIES; INTEREST;

AND ALL

PROPORTIONAL QUESTIONS
IN ONE RULE APPLICABLE TO THE WHOLE.

THE PROCESS GREATLY SIMPLIFIED AND ABRIDGED.

Ford

BY CHARLES G. BURNHAM, A. M.

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PREFACE.

The author of this Arithmetic has been engaged for some years past in the business of instruction, during a part of the time as a teacher of young ladies, and at other times as an instructor of pupils of both sexes. He has uniformly found, that there has been less interest excited in the study of Arithmetic than in most other branches of education.

5-20-38

In Rhetoric, Philosophy, Chemistry, Geography and History, there are many individuals in our schools who eagerly pursue their inquiries, while in Arithmetic this is rarely the case. Something may be set down to the circumstance, that the study of Arithmetic is more solitary in its character, and that the excitement is wanting that attends studies which are pursued more in classes. Arithmetic imparts fewer new ideas, and does not extend the range of thought and inquiry as much as many other intellectual pursuits. Its importance, as a mental discipline, and its practical advantages through life, are not sufficient to give interest to the scholar, and to beguile the tedium of a way that has few immediate attractions. Under such circumstances, the instructor will deem it highly important so to vary the mode of instruction as to render the study attractive, and to give that prominence to this department of education which its great importance demands.

The author of this work has labored earnestly, while in a course of instruction, to effect this object; and is satisfied, that the mode of instruction adopted in the work now offered to the public, possesses advantages in this respect over the systems of Arithmetic usually taught.

The process of abridging operations, by cancelling of numbers, has been in a few rules slightly alluded to in the systems of arithmetic now in use; but their authors do not seem to be aware of the extent to which it might be carried, in all proportional questions. Finding that this system gave new interest to the scholar; that it afforded an almost immediate solution to questions of the most difficult character; that it greatly abridged the labor of operation, in all cases where it could be introduced, and at all times carried satisfactory evidence of its perfect accuracy in principle, the author of this Arithmetic has expended much time and labor in extending this mode of operation to all arithmetical rules involving proportion, and in giving full explanations and demonstrations of its accuracy of application.

A number of teachers are now employed in teaching upon this principle, whom the author instructed to qualify them for this purpose; and they all concur in the advantages of the system.

The principle upon which cancelling proceeds is plain. It is merely taking from equal balanced scales equal weights, leaving the balance undisturbed. As applied in arithmetic, each process greatly simplifies the expression of the proportion.

In Fractions, $\frac{8}{16}$ is $\frac{4}{8}$, or $\frac{2}{4}$, or $\frac{1}{2}$. Here each term of the fraction is equally diminished, and the fraction remains the same in value, but the expression is greatly simplified. By stating a proportional question, and cancelling from it all redundancies, the answer is alone left, or so few portions of the statement remain, that, by a perfectly easy combination, in the manner presented by the rule, the answer is at once obtained.

There is an interest attending the statement of a question upon this system which does not occur in other modes. All fractions in the processes of Multiplication and Division are treated as whole numbers; the numerator being the same as the dividend in Division, and the denominator the same as the divisor. The facility of passing from one denomination to another, and of performing the reduction, when the terms are of different denominations, is of great advantage. The cancelling is an amusement, and the necessary combination of numbers, to close the question is ordinarily one of the simplest arithmetical operations. The scholar finds that the most difficult questions yield readily to his comprehension of them, and to this mode of solution, and he has the satisfaction of proceeding upon principles which he knows to be unerring.

The system of cancelling is applicable, as stated in the title page of this work, to all Proportional Questions, embracing the Rules of Three, Single and Double, Direct and Inverse; Interest; Discount; Barter; Loss and Gain; Exchange; Reduction, Multiplication and Division of Fractions, &c.

On the 129–30–31–35–36 and 137th pages,and in many other parts of the work, are questions shewing the mode of operation, as compared with the rules in ordinary use; some of these are selected from other works, for the purpose of illustration. On the 121—2—3—4 and 125th pages, are questions worked out, and the principle explained and illustrated.

Questions of the following kind are found in most Arithmetics, viz.

1. If 3 men can build 360 rods of wall in 24 days, how many rods can 8 men build in 27 days?

2. If a man can travel 240 miles in 12 days, when the days are 12 hours long, how far can he travel in 27 days, when the days are 16 hours long?

3. A merchant bartered $5\frac{8}{9}$ cwt. of sugar, at $6\frac{3}{4}$d. per pound, for tea at $8\frac{5}{8}$s. per pound. How much tea did he receive?

4. If 500 men consume $102\frac{5}{7}\frac{3}{4}$ barrels of flour in 9 months, how many barrels will 365 men consume in the same time?

These questions are wrought out on the pages first referred to; and all similar questions can be stated and solved by this system, with a facility vastly beyond that of proceeding by the ordinary rules. It is of peculiar importance in solving questions which arise in the daily transactions of business men; and the rules of the system are so simple, that when they have been once practiced, they can never be supplanted from the memory.

It would be useless to make these remarks, unless fully convinced of their accuracy. We are aware that in submitting this work to the public, so far as it is introduced, it would be a serious detriment to it, to claim for it any thing more than would be readily conceded by instructors and pupils.

It is submitted to them, with an earnest desire that it may meet their approbation, and with no other object than to aid in promoting this important branch of science.

A mere casual inspection of an arithmetic will of course furnish no evidence of its utility. It should be taken up systematically, and its mode of operation tested. We commend it to those who will bestow this labor upon it, and should regret to learn that they have reason to dissent from us.

The ordinary rules are inserted throughout, side by side, with the system of cancelling in those rules where that mode of operation is adopted; and it is believed that this portion of the work will compare favorably with other arithmetics. Many of the rules must be uniform in all systems, and no system of arithmetic can dispense with mechanical labor. All the primary rules preparatory to those involving proportions are in all cases essential, and the science of numbers in its best estate will require skill and labor.

Arithmetical computation and combination manifest a high advance in civilization. Barbarous nations have no idea of numbers beyond a very limited extent, and all results in science, obtained by calculation, are wrought out on the simple principles of arithmetic. Without her aid, Astronomy, Geography and Navigation would be a mere isolated detail of a few barren facts, and Commerce would be utterly at an end.

All conclusions, based upon relation of space and distance, till discoveries have been extended to the utmost verge of creation, have been achieved by arithmetic, attending as the handmaid of science.

Its power, as a means of high attainment and mental discipline, and its practical utility, should command attention to its study and interest in its instruction. It should be second to no other branch in either department of male or female education.

CHARLES G. BURNHAM.

PEMBROKE, N. H., OCTOBER 23, 1837.

The author of this work acknowledges himself indebted to Mr. HENRY L. BURNHAM, for many original arithmetical questions, and also for important aid in facilitating the preparation of the work for the press.

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EXPLANATION OF CHARACTERS USED IN THIS WORK.

$=$ Equality is denoted by two horizontal lines.

$+$ Addition; as $4+3=7$, which signifies that 4 added to 3 equals 7.

$\times$ Multiplication; as $4\times3=12$, which signifies that 4 multiplied by 3 equals 12.

$-$ Subtraction; as $3-4=1$, which signifies that 3 taken from 4 leaves 1.

$)($, $\div$, $\frac{4}{2}$, $_2|^4$, Division; as $2)4(2$ and $4\div2=2$, and $\frac{4}{2}=2$ and $_2|^4=2$; in either case it is signified that 4 divided by 2 equals 2.

$:\ ::$ Proportion; as $2 : 4 :: 6 : 12$; which is read, 2 is to 4 as 6 is to 12.

ARITHMETIC.

Arithmetic is the science of numbers. It explains their properties, and teaches how to apply them to practical purposes.

The principal or fundamental rules are, Notation, Numeration, Addition, Subtraction, Multiplication and Division. These are called fundamental rules, because all questions in Arithmetic are solved by one or more of them.

Notation is the expressing of any number or quantity by figures; thus, 1 one; 2 two; 3 three; 4 four; 5 five; 6 six; 7 seven; 8 eight; 9 nine; 0 cipher. Figures are sometimes called digits; they were formerly all called ciphers; hence, the art of Arithmetic was called ciphering.

There are two methods of Notation, the Arabic, as above, and the Roman, which is expressed by the following seven letters of the Alphabet.

I, V, X, L, C, D, M.

1 2 3 4 5 6 7 8 9 10 20 30 40
I, II, III, IV, V, VI, VII, VIII, IX, X, XX, XXX, XL.
50 60 70 80 90 100 500 1,000
L, LX, LXX, LXX, XC, C, D, M.

When a letter of less, is placed before one of a greater value, it diminishes the value of the greater, by the value of itself—thus, X signifies ten, but IX is only nine. When a letter of less, is placed *after* one of greater value, it increases the value of the greater by the value of itself.

This method is seldom used except in numbering chapters, sections, &c.

Questions. 1. What is Arithmetic? 2. What are the principal, or fundamental rules? 3. Why so called? 4. What is Notation? 5. What are figures sometimes called? 6. What were they formerly called? 7. How many methods of Notation, and what are they? 8. How many are the Arabic characters, or figures? 9. By what is the Roman method expressed? 10. How is a letter affected, when one of less value is placed before it? 11. How when one of less value is placed after it? 12. For what is the Roman method of notation principally used?

NUMERATION.

Numeration teaches to express in words the value of any number represented by figures. Thus, 365 is read, three hundred and sixty-five.

Figures have a simple and relative value. When a figure stands alone, its value is simply so many units, or ones; as, 2 two; 3 three; 4 four. Their relative value is derived from the place they occupy when joined together, or from their distance from the unit's place. Thus, 2 and 3 express their own value; simply so many units; but they are made to express either 23 or 32; that is, either three units and two tens, or two units and three tens. Hence, it appears, that the first or right hand place always expresses so many units; it is therefore called the unit's place; the second, the place of tens, expressing always as many tens as the figure contains units. The third place is hundreds—the fourth thousands, as may be seen by the following

TABLE.

Hundreds of millions	Tens of millions	Millions	Hundreds of thousands	Tens of thousands	Thousands	Hundreds	Tens	Units	
								1	One
							2	1	Twenty one
						3	2	1	Three hundred and 21
					,4	3	2	1	Four thousand and 321
				5	,4	3	2	1	Fifty four thousand and 321
			6	5	,4	3	2	1	Six hundred and 54 thou. 321
		,7	6	5	,4	3	2	1	7 millions 654 thou. 321
	8	,7	6	5	,4	3	2	1	87 millions 654 thou. 321
9	8	,7	6	5	,4	3	2	1	987 millions 654 thou. 321

The cipher, when standing alone, or at the left hand of another figure, signifies nothing, as 05, 005, is five in either case, because it still occupies the unit's place. But when placed at the right hand of another figure, it increases its value in a tenfold ratio, by removing the figure farther

---

Questions. 13. What is Numeration? 14. What is the value of a figure standing alone? 16. From what is their relative value derived? 17. What does the first, or right hand figure, always express, and what is it called? 18. What is the value of the cipher, when standing alone, or at the left hand of another figure? 19. What effect has it when placed at the right of another figure?

from the unit's place. This may be seen by the following—

TABLE II.

0	Nothing.
20	Twenty.
200	Two hundred.
2000	Two thousand.
20,000	Twenty thousand.
200,000	Two hundred thousand.
2,000,000	Two millions.

To know the value of any number of figures.

Rule 1. Numerate from the right hand to the left, by saying units, tens, hundreds, &c., as in the Table.

2. To the simple value of each figure join the name its place, reading from the left hand to the right.

TABLE III.

Duodecillions	Undecillions	Decillions	Nonillions	Octillions	Septillions	Sextillions	Quintillions	Quadrillions	Trillions	Billions	Millions	Thousands	Units
783	694	542	987	562	714	923	610	782	184	542	365	987	963
Period of Sextillions		Period of Quintillions		Period of Quadrillions		Period of Trillions		Period of Billions		Period of Millions		Period of Units	

The first division of the foregoing Table is according to the French method, into periods of three figures each; the name of the period is superadded. The second division is according to the English method, into periods of six figures each; the name of each period is subjoined. The two divisions of the Table agree for the first nine figures—beyond that they assume different names. The principles of Notation in both are the same. In the former method, the names, units, tens, hundreds, are repeated in each period; in the latter method, thousands, tens of

---

Questions. 20. How may the value of any number be found? 21. What are the two methods of numeration in the third table? 22. In what respect do they differ?

thousands, hundreds of thousands, are repeated with the name of the period.

Let the scholar point the following numbers into periods and read them.

3445
67891
983452
5437643
67821356
436543897
5678923412
96754329876
1234678901268

Express the following numbers in figures.

1. Twenty-three.
2. Thirty-five.
3. One hundred and twenty.
4. One hundred and twenty-six.
5. Ten thousand three hundred and twenty.
6. One million two hundred and twenty thousand, three hundred and forty-five.
7. Two billions, twenty seven millions, three hundred and forty thousand, four hundred and seventeen.
8. Forty five quadrillions, six hundred and twenty trillions, four hundred and twenty billions, four hundred and twenty millions, two hundred and twenty thousand, three hundred and nineteen.

The scholar should be well versed in Notation and Numeration, before proceeding to the following questions.

EXERCISES.

1. If John has 6 apples, and his brother gives him 3 more, how many will he have?

2. James being on a visit at his uncle's, one of his cousins gave him 3 walnuts, another 4, and his uncle gave him 9, how many did he receive, and how many more did his uncle give him than his cousins?

QUESTION. 23. In what should the scholar be well versed before proceeding any further?

3. Samuel bought a book for 15 cents, and sold it for 17, how many cents did he gain?

4. If a boy pay 15 cents for a book, 10 for a knife, and 6 for a dozen apples, how many cents does he pay in all, and how many more for the book than for the knife, and how many more for the book and knife than the apples?

5. If an inkstand cost 10 cents, an orange 5, a lemon 3, and a dozen of quills 14 cents, what is the cost of the whole, and how much more will the inkstand, orange and lemon cost than the quills?

6. A man bought of a drover 3 sheep and a cow; for one of the sheep he paid 4 dollars, for the other two he paid 3 dollars apiece, for the cow he paid 20 dollars; how many dollars did he pay for the whole, and how much more did he pay for the cow than for the sheep?

7. Joseph bought a sled for 25 cents, a yoke for 12 cents, and a whip for 6 cents; what did they all cost him, and how much more did the sled cost than the yoke and whip?

8. If I pay 6 dollars for a hat, 8 for a cap, 4 for a vest, and 14 for a coat, what do I pay for the whole, and how much more do the coat and vest cost, than the hat and cap?

9. If I owe one man 6 dollars, another 8, another 12, another 20, how much do I owe in all?

10. The scholars in a certain school are divided into 4 classes—in the first class there are 10 scholars, in the second 12, in the third nine, and in the fourth 14, how many in all?

11. If from my library, I lend to one man 5 books, to another 10, to another 8, to another 12, to another 20, how many do I lend in all?

12. In my garden there are 6 apple-trees, 8 pear-trees, 10 peach-trees, 18 plum-trees, how many trees are there in all?

13. In a certain school, 10 study music, 12 French, 14 Spanish, how many are there, and how many more study French and Spanish, than music?

14. Eliza had 4 finger-rings, Mary had 10, and Susan had 7: how many more had Mary than Eliza, and Susan than Mary, and how many had they in all?

15. A certain man had 4 boarders; for two he received

3 dollars each per week, for one 2 dollars, for another 5: how much did he receive per week?

16. A young lady bought 2 dresses; for one she paid 7 dollars, for the other, 13 dollars; how much more did the last cost than the first, and how much did she pay for both?

Signs.—A cross + is the sign of addition. It shows that the numbers between which it is placed are to be added. Two parallel horizontal lines = signify equality. Thus 3+4=7 is read, 3 added to 4, or 3 plus 4 *(plus is a Latin word, which signifies more)* is equal to 7. A short horizontal line — signifies subtraction. Thus 7—4=3 is read 7 minus, *(minus is a Latin word, which signifies less)* 4 is equal to 3.

ADDITION AND SUBTRACTION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	5	6	7	8	9	10	11	12	13	14
3	5	6	7	8	9	10	11	12	13	14	15
4	6	7	8	9	10	11	12	13	14	15	16
5	7	8	9	10	11	12	13	14	15	16	17
6	8	9	10	11	12	13	14	15	16	17	18
7	9	10	11	12	13	14	15	16	17	18	19
8	10	11	12	13	14	15	16	17	18	19	20
9	11	12	13	14	15	16	17	18	19	20	21
10	12	13	14	15	16	17	18	19	20	21	22

Explanation of the foregoing table for addition.—Look for the number to be added, on the left hand, and, following the horizontal line towards the right, the amount will be found on this line directly under the number to which it is to be added. If it be required to add 6 to 8, look for 6 on the left, and, following the line upon which 6 stands, directly under 8 will be found 14, the amount required.

The same table may be used for subtraction. Look for the number to be subtracted, on the left hand, and follow

---

QUESTIONS. 1. What is the sign of Addition? 2. Sign of equality? 3. Sign of Subtraction? 4. What does plus signify? 5. What does minus?

the horizontal line, until you find the number from which it is to be subtracted, and directly over this number, at the top of the line, will be found the difference between the numbers.

ADDITION.

Simple Addition is the putting together of two or more numbers of the same name or kind, to find the amount.

Note. It is called Simple Addition, because the numbers are all of one denomination; that is, all dollars, or all cents. When the numbers are pounds, shillings, pence, &c., the denominations are different.

If one man owe me 25 dollars, another 22, to find the amount of what both owe, I write the sums in the following manner, units under units, tens under tens, and add them together, thus:

$$\begin{array}{r} 25 \\ 22 \\ \hline 47 \end{array}\ \text{amount.}$$

Beginning at the right hand, or units place, I say 2 and 5 are 7; then, in the second place, or place of tens, I say 2 and 2 are 4—which is four tens, or 40.

2. A man has three fields; one contains 31 acres, another 25, another 42; how many acres are there in all?

Operation.

$$\begin{array}{r} 31 \\ 25 \\ 42 \\ \hline \end{array}$$

Ans. 98

Having written the numbers according to the directions, units under units, tens under tens, &c., we begin at the right hand to add, and find the amount to be 8 units, which we place under the column of units. The amount of the second column, or column of tens, we find to be 9 tens, or 90. The answer, then, is 9 tens and 8 units, or 98.

3. What will a carriage, horse and harness cost, if the carriage cost 102 dollars, the horse 80 dollars, and the harness 16 dollars? *Ans.* 198.

4. If a waggon cost 78 dollars, and a yoke of oxen 96 dollars, what will be the cost of both?

In the preceding examples, the numbers when added

Question. 1. What is Addition?

have been less than 10, and of course have required but one figure to express them. In the last column, it will be seen that the numbers in the unit column, when added, amount to more than 10, and in the column of tens the amount is more than 10—that is, ten tens.

Illustration. 78
96
—
14
16
——
Ans. 174

Placing the numbers as before directed, and adding the right hand column, we find it amounts to 14 units, or 1 ten and 4 units. The next column amounts to 16 tens, or 100 and 6 tens, which, when added, make 4 units, 7 tens, and 100; or 174—the answer. This mode of writing numbers in large sums would be long and tedious; it is, therefore, better to carry on a part of the operation in the mind, thus:

78
96
——
174

Six and eight are fourteen; we place the 4, or excess of ten, in the unit column, and add the 1 ten to the column of tens. The column of tens added together with the ten from the unit column, makes 17 tens. The answer, then, is 17 tens and four units, or 174. From the above, it is evident, that one in the column of tens is equal to ten in the column of units, and one in the column of hundreds is equal to ten in the column of tens.

RULE.

Add each column, beginning at the right hand, and set down the amount directly under the column, if it be less than 10; but if it be 10 or more, set down the right hand figure and add the left to the next column, and there set down the whole amount. This is the same as carrying one for every ten.

Proof. Perform the addition downwards, or, cut off the top line, and find the amount of the remainder; add this amount to the top line, and if this last amount correspond with the sum total, the work is supposed to be right.

EXAMPLES.

1. If a man pay 1496 dollars for a house, 734 dollars for a lot of land, 300 dollars for rail-road stock, and 145

QUESTIONS. 2. What is the rule for Simple Addition? 3. How is Addition proved?

dollars for a share in a bridge, how much does he expend in the whole? *Ans.* 2675 dollars.

2. A man sold plank to the amount of 834 dollars; boards to the amount of 376; shingles to the amount of 400; timber 621; two masts, one for 30 and the other for 50 dolls.: what was the amount of the whole?

Answer, 2311 dollars.

3. A merchant on settling his accounts finds himself in debt to A. $100; B. 60; to D. 78; to C. 80; to E. 447; how much does he owe in all? *Ans.* 765 dolls.

4. From the creation of the world to the christian era was 4004 years; from that time to the declaration of American Independence was 1776, and 61 years since that period. How many years since the Creation?

Answer, 5841 years.

```
1 2 6 7 3 1 6 2 3 8 1
7 6 1 3 4 8 4 7 5 3 8
8 0 4 8 1 9 0 5 4 6 4
3 5 4 9 7 2 4 3 0 7 7
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```

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6 8 9 7 4 3 2 7 1 8
1 1 1 8 6 5 3 1 2 7
5 7 3 4 8 6 9 2 5 6
8 1 6 0 0 0 0 0 8 2
    5 4 1 2 1 7 8 3
      3 4 1 6 9 3 4
      0 1 5 7 6 0 9
        3 6 8 9 1 2
-------------------

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EXERCISES.

1. A man by his will left his two sons 1450 dolls. each; his four daughters 1200 each; to his wife 1500, to various charitable objects 1834, what was the value of his estate?

Answer, 11034 dollars.

2. If 1889 figures cover one side of a slate, how many will it take to cover both sides of 4 slates? *Ans.* 15112.

3. Bonaparte was born in the year 1769, lived 52 years. In what year did he die? *Ans.* 1821.

4. General Jackson took the Presidential chair in 1829, occupied it 8 years. In what year did his course termiminate. *Answer*, 1837.

5. George Washington was born in the year 1732. He lived 67 years. In what year did he die? *Ans.* 1799.

6. The distance from New-York to Rahway, N. J. is 20 miles, from Rahway to New-Brunswick 12 miles, from N. B. to Princeton 18 miles, from Princeton to Trenton 12 miles, from Trenton to Bristol 10 miles, from Bristol to Philadelphia 20 miles. What is the distance from New-York to Philadelphia. *Answer*, 92 miles.

7. Lafayette was born in the year 1757. He died at the age of 78. In what year did he die? *Ans.* 1835.

8. A man sold five oxen, each weighing 864 pounds, how much did they all weigh? *Ans.* 4320.

9. How many times does a common clock strike in 24 hours? *Ans.* 156.

10. A gentleman left his two sons each 1480 dollars, his only daughter 1500 dollars, and his wife 200 more than all his children; what was the wife's portion, and what was the value of the whole estate?

Ans. Wife's portion 4660 } dolls.
Whole estate 9120 }

11. There are two numbers, the less is 1768, their difference is 961, what is the larger number?

Ans. 2729.

12. From Boston to Providence it is 40 miles; from Providence to New-York 198 miles; from New-York to Philadelphia 92 miles; from Philadelphia to Wilmington 28 miles; from Wilmington to Baltimore 72 miles; from Baltimore to Richmond 110 miles; from Richmond to Raleigh 155 miles; from Raleigh to Charleston 256 miles; from Charleston to Savannah 113 miles; from Savannah to New-Orleans 713 miles; how many miles from Boston to New-Orleans, passing through the above places?

Answer, 1777 miles.

13. A man bought five firkins of butter, one firkin contained 150 pounds, another 60, another 75, another 98, another 125. How much did they all contain? *Ans.* 508.

14. There were five churches erected, one in ——, which cost 16,500 dollars, two in ——, which cost 18,350 dollars each, one in ——, which cost 19,386 dollars, and one in ——, which cost 12,640 dollars. How much was the expense of the whole? *Ans.* 85,226 dollars.

When the columns to be added are long, the following method will be found convenient. Begin to add with the

---

QUESTION. When the columns to be added are long, how may you proceed?

unit figure as usual; and for every ten, place a dot against that figure which makes ten or more than ten, and add the excess to the figure above it, and thus proceed to the top of the column. Write the excess of ten at the foot of the column added, then count the dots, and as many as they are, so many carry to the next left hand column.

27687	3978	3 4.9.5.
78989	2129	6.7 8.9.
77896	9123	2 3.1 6
98987	5769	6.1 2 5.
98987	4320	7 6 3.4
65769	9621	2.3.1 6.
69776	3978	9 8 2 0
75645	8732	4.5.6.7.
———	1256	2 1 4 8
	———	———
		45210

SUBTRACTION.

1. John's father gave him 6 apples. He gave his brother 4 of them. How many had he left?

2. Joseph bought sixpence worth of candies and ninepence worth of hazelnuts. How much more did he give for the hazelnuts than for the candies?

3. Henry was 10 years old when his mother died, his sister was 6. How much older was Henry than his sister?

4. *Carolus Good Luck*, as he was going to school, found a purse containing 10 dollars. As a reward for his honesty in faithfully delivering it to the owner, he received 3 dollars; how many were there left?

5. Peter's father gave him 10 cents. As he was going to school he found 25 more; but, for dishonesty in trying to conceal his prize from the rightful owner, his father took from him the 10 cents which he gave him. How much more did he find than he lost?

6. Timothy bought a slate for 18 cents, and a spelling book for 28 cents, how much more did he give for the book than for the slate?

7. Susan had 6 frocks; 4 of them she burnt; how many had she left?

SIMPLE SUBTRACTION,

Teaches to find the difference between two numbers of the same name or kind. (See note p. 19.)

To perform the operation, two numbers are required, the larger is called the Minuend, the less the Subtrahend; the difference between them is called the Remainder.

RULE.

1. Place the numbers, the less under the greater; units under units, tens under tens, hundreds under hundreds, &c. Begin at the right hand, or unit's place, and take each figure in the lower line from the one above it, and set down the remainder. If either of the lower figures be greater than the one above it, suppose ten to be added to the upper figure, subtract the lower figure from it, and set down the difference, observing to carry one to the next left hand figure.

Proof.—Add the remainder and lower line together; if the work be right, the amount will correspond with the upper line.

EXAMPLES.

	Operation.
From	3452
Take	2561
Remainder	0891
Proof.	3452

Beginning at the right hand, we say, 1 from 2 and 1 remains, which we set down; then, because 6, the next left hand figure, cannot be taken from 5, the figure above it, we suppose 10 to be added to 5, which is 15; then 6 from 15 and 9 remain, which we set down. We now carry 1 to 5, the next figure, because we borrowed 10, and say, 6 from 14, borrowing 10 again, and 8 remain. And, lastly, 1 to carry to 2 is 3—3 from 3 and nothing remains.

QUESTIONS. 1. What does Simple Subtraction teach? 2. Why is it called simple? 3. To perform the operation, how many numbers are required? 4. Which is the minuend? 5. Which is the subtrahend? 6. What is the remainder? 7. What is the rule for simple subtraction? 8. How do you prove subtraction?

From	39670	From	506789	From	67023491
Take	28931	Take	467898	Take	57216532

5. From four hundred and seventy-nine, take three hundred and seventy-five.

6. Take twenty-five thousand, nine hundred and twenty-three, from forty-four thousand, five hundred and twenty.

7. What number must be subtracted from 2081 that the remainder may be 1104? *Answer*, 977.

8. From thirty-four thousand, take seventeen thousand and ninety-one.

EXERCISES.

1. John's uncle gave him 20 cents. He lost 5 of them; how many had he left?

2. In a school of 48 scholars, 21 left; how many remained? *Answer*, 27.

3. A western hunter met with 45 buffaloes in one drove—killed all but 18; how many did he kill? *Ans.* 27.

4. Sir Isaac Newton was born in the year 1642, and died in 1727; how old was he when he died? *Answer*, 85 years.

5. The mariner's compass was invented about the year 1302; how long after that period was Sir Isaac Newton born? *Answer*, 340 years.

6. The first settlement in New-England was made at Plymouth, by the Puritans, in the year 1620; how long is it since that time, to the year 1837? *Ans.* 217 years.

7. Gun-powder was invented in the year 1330. How long was it after the invention of the mariner's compass? *Ans.* 28.

8. Virginia contains 64000 square miles; New-York contains 46000. What is the difference? *Ans.* 18000.

9. The library of Dartmouth College contains 12,800 volumes; Harvard University contains 34,600 volumes. How many does one contain more than the other? *Answer*, 21800.

QUESTIONS. 9. In the first example how do you subtract 6 from 5? 10. How do you pay the 10 which you borrowed? 11. Why is the 1, which you carry, equal to the 10, which you borrowed? *Answer.* Because 1, in the left hand column, is equal to 10 in the right.

10. Dartmouth College was incorporated in the year 1769, Harvard University in the year 1638. What is the difference in time? *Answer*, 131 years.

11. The population of the State of New-York, in 1820, was 1,372,812; in the year 1835 it was 1,616,482. How many years between these two periods, and how much was the increase? *Answer*, 15 years.
Increase, 243,670.

12. An officer with a company of 102 soldiers, was met by a party of Indians, who killed all his army but 17 men. How many were killed? *Answer*, 85.

13. A merchant bought 40 tons of wine, containing 10080 gallons, which cost him 2410 dollars. He sold 28 tons, containing 7056 gallons, for 1814 dollars. How many gallons had he left, and how much money did he want to make up the first cost? *Ans.* 3024 galls.
596 dolls.

ADDITION.	SUBTRACTION.
1. If a harness is worth 18 dollars, and the horse is worth 68 dolls. more than the harness, what is the value of the horse?	2. If a horse cost 86 dollars, and the harness 18 dollars, how much more than the harness did the horse cost?
3. If a merchant have 1734 yards of cloth, after selling 6588 yds., how many had he at first?	4. A merchant having 8322 yds. sells 6588 yds.; how many has he left?
5. What is the amount of 2269+8625?	6. What number must be added to 8625 to make 10894?
7. Dr. Franklin was born in 1706,—93 years before the death of Washington. In what year did Washington die?	8. How many years before the death of Washington, in 1799, was the birth of Franklin?
9. The mariner's compass was invented in 1302; Sir Isaac Newton was born 28 years after. In what year was he born?	10. Sir Isaac Newton was born in 1330; the mariner's compass was invented 28 years before. In what year was it invented?
11. If a piece of land be bought for 550 dolls., and is	12. If a piece of land sell for 800 dolls., which is 250

sold for 250 more than it cost, for how much was it sold?

13. Peter the Great died in 1725—112 years before the independence of Texas was acknowledged by the United States. In what year was the independence acknowledged?

15. Supposing a man to be born in the year 1738; lived 98 years; in what year did he die?

17. Columbus first sailed for America in the year 1492; the independence of America was declared 284 years after. In what year was it declared?

more than it cost, what was the first cost?

14. Peter the Great died in the year 1725. How many years from that period to the acknowledging of the independence of Texas, in 1837?

16. Supposing a man to be 98 years old in the year 1836, in what year was he born?

18. The independence of America was declared in the year 1776; 284 years before Columbus first sailed for America; in what year did he sail?

PRACTICAL QUESTIONS IN ADDITION AND SUBTRACTION.

1. Add 900, 400, and 752; subtract from their sum 647. *Answer*, 1405.

2. Charles had 18 peaches. He gave his mother 6 and his sister 4. How many had he left. *Ans.* 8.

3. A man buys at one store 84 eggs, at another 4 dozen, at another 3 dozen. As he returns, he sells 5 dozen. How many did he purchase, and how many had he left, when he arrived at home? *Answer*, 168—108.

4. James owes A 20 cents; B 30; C 40; D 50.—John owes A 18 cents; B 23; C 35, D 47. How much do they both owe, and which owes the most?

A merchant owes A 1300 dollars; B 1900 dollars; C 2500 dollars. He is worth 3500 dollars. How much does he owe, and how much more than he is worth?

6. A man borrowed of his friend at one time 100 dolls., at another 150 dolls., at another 175 dolls. He paid 275

dolls. How much did he borrow, and how much does he still owe? *Answer.* Borrowed 425.
Owes 150.

7. On a certain farm there are 750 apple-trees, 425 pear-trees, 1000 peach trees, and 389 plum-trees. What is the amount of the whole, and how many more apple-trees than pear-trees, and how many more peach than plum?

8. A man left to his wife 2500 dollars, to his four sons 900 dollars each, to his three daughters 450 each. What was the amount of property left, and how much more was left to the mother than to the daughters, and to the sons more than to the mother?

9. There were five important events in the course of 215 years, viz.—1. The invention of the mariner's compass. 2. The invention of gun-powder. 3. The art of printing. 4. The discovery of America. 5. The reformation. The last was accomplished, A. D. 1517; the third, 77 years before; the second, 42 years after the first, and the fourth 148 years after the second. The question is, In what year did each happen?

Answer. Mariner's compass in 1302, gun-powder in 1344, Printing 1440, Discovery of America, 1492.

MULTIPLICATION.

1. If Mary give 4 cents for one picture-book, how much must she give for 2 books? how much for 4? how much for 5?

2. If one dozen of eggs cost 10 cents, how many cents will two dozen cost? how many will 4? 5? 6?

3. If one share in a library cost 5 dollars, how much will three shares cost? how much 4? how much 5? how much 6?

4. If a picture frame cost 12 dollars, what will 4 cost? what will 6? what will 7? what will 8? what will 9?

5. Four men bought a piece of land, each paying 12 dollars. What did they all pay?

6. If a horse can trot 11 miles in one hour, how many miles can he travel in 8 hours? how many in 10? in 12?

7. If a bushel of wheat cost 2 dollars, how many dollars will 8 bushels cost? how many will 9? how many will 10? how many will 11? how many will 12?

8. If a man receive 4 shillings for a day's work, how much will he receive for a week's work?

9. A pound of sugar is worth 8 cents. What are 6—7—8—9—10—11—12 pounds worth?

10. John bought a writing-book for 6 cents. What will 2 cost? what will 4? what will 6? what will 8?

11. If a bushel of potatoes cost 32 cents, what will 3 bushels cost? what will 4? what will 6?

12. If a steam-boat sail 8 miles in one hour, how many will she sail in 12 hours?

The scholar should commit to memory the following TABLE before proceeding any further.

MULTIPLICATION AND DIVISION TABLE.

TWICE		3 TIMES		4 TIMES		5 TIMES		6 TIMES		7TIMES	
1make	2	1 make	3	1 make	4	1 make	5	1make	6	1make	7
2	4	2	6	2	8	2	10	2	12	2	14
3	6	3	9	3	12	3	15	3	18	3	21
4	8	4	12	4	16	4	20	4	24	4	28
5	10	5	15	5	20	5	25	5	30	5	35
6	12	6	18	6	24	6	30	6	36	6	42
7	14	7	21	7	28	7	35	7	42	7	49
8	16	8	24	8	32	8	40	8	48	8	56
9	18	9	27	9	36	9	45	9	54	9	63
10	20	10	30	10	40	10	50	10	60	10	70
11	22	11	33	11	44	11	55	11	66	11	77
12	24	12	36	12	48	12	60	12	72	12	84

8 TIMES		9 TIMES		10 TIMES		11 TIMES		12 TIMES	
1 make	8	1 make	9	1 make	10	1 make	11	1 make	12
2	16	2	18	2	20	2	22	2	24
3	24	3	27	3	30	3	33	3	36
4	32	4	36	4	40	4	44	4	48
5	40	5	45	5	50	5	55	5	60
6	48	6	54	6	60	6	66	6	72
7	56	7	63	7	70	7	77	7	84
8	64	8	72	8	80	8	88	8	96
9	72	9	81	9	90	9	99	9	108
10	80	10	90	10	100	10	110	10	120
11	88	11	99	11	110	11	121	11	132
12	96	12	108	12	120	12	132	12	144

13. If a man pay 85 dollars for a carriage, what must he pay for 5 carriages?

The answer may be obtained by setting down 85, five times, and adding them up, thus:

85
85
85
85
85
———
425
———

It will be seen by examining this operation, that the product of five times five units is two tens and five units,—and five times eight tens is 40 tens. The answer, then, is 40 tens, 2 tens and 5 units, or 425. This method, in long sums, would be tedious, and can be resolved much easier by multiplication, thus:

85
5
———
425

Instead of setting down 85, five times, we write 5, the muitiplier, under the unit figure of the number to be multiplied; then say, 5 times 5 are 25, setting down 5, the excess of tens, and reserving, in the mind, 2, the number of tens; we say 5 times 8 are 40, adding the two tens which we reserved from the unit column, we set down 42. The answer, then, is 42 tens and 5 units, or 425.

From the above we derive the following definition:—*Multiplication* is a concise method of performing many additions. Two numbers are required to perform the operation. The number to be multiplied is called the Multiplicand. The number by which we multiply is called the Multiplier. Taken together they are called Factors. The answer obtained is called the Product.

14. If a share in a bridge is worth 142 dollars, how many dollars are 6 shares worth?

Operation.

142
6
———
852 *Answer.*

QUESTIONS. 1. What is Multiplication? 2. How many numbers are required to perform the operation? 3. What is the number to be multiplied called? 4. What is the number by which you multiply, called? 5. Taken together, what are they called? 6. What is the answer called? 7. How many figures are there in the multiplier of the 14th question?

15. If a man receive 164 dollars for a year's labour, what will 32 men receive for the same time?

```
  164
   32
 ----
  328
 492
 -----
 5248
 -----
```

12 being the largest number by which we can multiply collectively, it will be necessary in this example to adopt a different mode of operation. The multiplier consists of 3 tens and 2 units, and multiplying by the 2 units, we obtain for a product, 328. We then multiply by the three tens, setting the product, 2, in the place of tens, because it is the product of tens. We next multiply the 6 tens of the multiplicand by the three tens of the multiplier, carrying one for every ten,as in Addition. The product of ten multiplied by ten is hundreds; we, therefore, set it down in the place of hundreds. Lastly: we multiply the hundreds in the multiplicand by the tens in the multiplier, and set the product in the place of thousands. The product of hundreds multiplied by ten is thousands. Adding together the several products we have 5248, the answer.

From the foregoing examples we derive the following

RULE.

I. Place the multiplier directly under the multiplicand, units under units, tens under tens, &c., then draw a line underneath.

II. When the multiplier is 12, or less than 12, begin at the right hand of the multiplicand, and multiply each figure contained in it by the multiplier, setting down the numbers, and carrying as in Addition.

III. When the multiplier is greater than 12, write down the figures as before directed, and multiply the multiplicand by each figure in the multiplier, commencing with the unit figure; observing to place each figure in the product directly under the figure by which you multiply. In this way proceed, and the sum of the products will be the answer.

QUESTIONS. 8. By which do we multiply first? 9. What is the product of units multiplied by units? 10. Of tens multiplied by tens? 11. Of hundreds by hundreds? 12. How are the numbers placed in multiplication? 13. How do you proceed when the multiplier is 12 or less than 12? 14. When the multiplier is greater than 12?

EXAMPLES.

3467	124567	54678901	34567892
6	8	341	4567
679834	678345	126789123	908764584768
12	23	27678	632976

PRACTICAL EXAMPLES.

9. What will 432 barrels of flour cost, at 14 dollars a barrel? *Answer*, 6048 dollars.

10. How many rods in 84 miles, there being 320 rods in a mile? *Answer*, 26880 rods.

11. What will be the cost of 6328 thousands of boards, at 18 dollars per thousand? *Ans.* 113904 dollars.

12. What will 64 cows cost, at 16 dollars apiece? *Answer*, 1024 dollars.

The multiplier, 16, in the last example, is a number which can be formed by the multiplication of two numbers —thus: 4×4=16: or 8×2=16. Any number thus produced is called a composite number. The numbers thus multiplied are called component parts. Sixteen, then, is a composite number, and 4 & 4, or 8 & 2 are the component parts of 16. Thus, taking the above question, 64×4 = 256, the price of 64 cows at 4 dollars each, and this product multiplied by 4 gives 1024, the price at 16 dollars each, because 4 times are 16. The same result will be produced if we multiply 64 by 8 and 2.

RULE.

When the multiplier is a composite number, multiply first by one of the component parts, and that product by the other, and so on, if there be more than two; the last product will be the answer.

QUESTIONS. 15. What is a composite number? 16. What are the numbers called which form a composite number? 17. What is the rule for multiplying by a composite number?

13. What is the product of 78 multiplied by 25?

78
5

It will be seen that 5 and 5 are the component parts of 25.

390
5

1950 *Ans.*

14. There are 365 days in a year. If a man live 48 years, how many days has he lived? *Ans.* 17520.

15. Multiply 7684 by 112. 8×7×2=112.
16. — 8410736 by 56.
17. — 17548671 by 81.
18. — 998673214 by 1864.
19. — 99998887777 by 445566.
20. — 88900236789456 by 77889123.

21. If it take 142 stones to build a rod of wall, how many will it take to build 10 rods?

142
10

1420

It will be seen that the answer in this example is the same as the multiplicand, with the addition of one cipher. If, then, we annex a cipher to the multiplicand, it is the same as multiplying by 10; since annexing a cipher to the right hand of any number increases the value tenfold. We have, then, this rule for multiplying by 10, 100, 1000, or 1 with any number of ciphers. Annex as many ciphers to the multiplicand as there are ciphers at the right hand of the multiplier, and it will give the answer required.

22. Multiply 142 by 100. *Ans.* 14200.
23. — 864 by 1000. *Ans.* 864000.
24. — 999 by 10,0000. *Ans.* 9990000.
25. — 2400 by 2200. *Ans.* 5280000.

Operation. 2400
2200

48
48

5280000

In this example, we multiply by the significant figures only, placing as many ciphers at the right hand of the product as there are ciphers in the multiplier and multiplicand.

26. What is the product of 68400 multiplied by 18000? *Ans.* 1231200000.

QUESTIONS. 18. When the multiplier is 10, 100, 1000, &c. how may you proceed? 19. When there are ciphers at the right hand of the multiplier and multiplicand?

When there are ciphers between the significant figures, reject them, observing to place the first product of every figure directly under that figure—thus: *Operation.*

27. Multiply 2008 by 604.

```
 2008
  604
-----
 8032
12048
-------
1212832
```

28. Multiply 8624 by 108.

29. — 340824 by 909.

30. — 5678902 by 770901.

31. What will 412 hogsheads of molasses cost at 31 dollars per hhd.? *Ans.* 12772 dolls.

32. What number is that of which 8, 9, 11, are factors? *Ans.* 792.

33. If 80 men dig a canal in 92 days, how many men could dig the same in one day? *Ans.* 7520.

34. If they receive 5 shillings per day, how many shillings do 7520 men receive? *Ans.* 37600.

35. A merchant bought 28 boxes of sugar, each weighing 235 lbs., at 8 cents per lb. How many cents did they cost? *Ans.* 52640.

36. How many shillings will 89 cords of wood cost, at 15 shillings per cord? *Ans.* 1335.

37. A merchant bought 15 pieces of cloth, each piece containing 27 yards at 7 dollars per yard. How much did he pay for the whole? *Ans.* 2835 dollars.

38. If a ship sail 12 miles per hour, how far will she sail in 12 days? *Ans.* 3456 miles.

39. If a man hoe 3 rows, 28 hills each, in one hour, how many hills will he hoe in 12 days, working 8 hours in a day? *Ans.* 8064.

40. What will 50 firkins of butter cost, weighing 54 lbs. each, at 14 cents per lb. *Ans.* 37800.

42. A man has 9 piles of wood, 16 cords in a pile. What is it worth at 7 dollars per cord? *Ans.* 1008 dolls.

QUESTIONS. 20. How do you proceed when there are ciphers between the significant figures?

DIVISION.

1. John has 5 oranges given him. He keeps one himself, and divides the others equally between his two sisters. How many did each receive?

2. Samuel divided 9 walnuts equally between three boys. How many did each receive?

3. James' father gave him 8 butternuts to divide equally between himself and his 3 brothers. How many did each receive?

4. If Harriet gave 24 cents for 6 pictures, what did she pay for 1?

5. Mary divided 36 cents equally between 6 poor children. What did each receive?

6. If 8 yards of cloth cost 56 cents, what did one cost?

7. How many yards of broadcloth can be bought for 72 dollars, at the rate of 8 dollars per yard?

8. How many bushels of apples can be bought for 100 cents, at 25 cents per bushel?

9. How many barrels of flour may be bought for 77 dollars, at 7 dollars per barrel?

10. In how many hours will a man travel 48 miles, at the rate of 4 miles per hour?

11. Eighty cords of wood piled in 8 different piles. How many cords in each pile?

12. A farmer sold wool to the amount of 81 dollars, for 9 shillings a fleece. How many fleeces did he sell?

13. How many thousand of boards may be bought for 144 dollars, at 12 dollars per thousand?

14. How many books may be bought for 84 cents, at 7 cents apiece?

15. If 16 apples be divided equally between 4 boys, how many does each receive?

It is evident, that, as many times as 4 is contained in 16, or, as many times as it can be subtracted from it, so many apples each boy will receive, thus:

$$\begin{array}{r} 16 \\ \underline{4} \\ 12 \\ \underline{4} \\ 8 \\ \underline{4} \\ 4 \\ \underline{4} \\ 0 \end{array}$$

We find by trial, that 4 is contained in 16 four times, which is the number of apples each boy is to receive.

If 4 boys receive 4 apples each, how many do they all receive?

It is plain that 4 boys will receive 4 times as many as one; therefore, if one boy receive 4 apples, 4 boys will receive $4\times4=16$. From this example, it is evident that Division is a concise method of performing many subtractions, or, the reverse of Multiplication. As in multiplication 2 numbers are required to perform the operation, so in division. The number to be divided is called the Dividend; the number by which you divide is called the Divisor. The Dividend is to be regarded as the product of two factors, of which the Divisor is one, and the other is sought, which is the Quotient after division. The divisor and quotient multiplied together produce the dividend. Thus, it appears that Division and Multiplication mutually prove each other.

SHORT DIVISION.

16. If 336 dollars be divided equally among 3 men, how many dollars will each receive?

To subtract 3 from 336 as many times as would be necessary to give each man his share, would be long and tedious; but, by Short Division the operation becomes simple: 336 is 3 hundreds, 3 tens, and 6 units. It will be perceived, that, if 300 be divided into 3 equal parts, one of these parts will be 100, and three tens divided in like manner becomes 1 ten, and 6 units divided by 3 become 2 units. The answer, then, is 1 hundred 1 ten and 2 units, or 112.

300 divided by 3 gives 100, 30 divided by 3 gives 10, and 6 divided by 3 gives 2: Then $100+10+2=112$ *Answer.*

Operation.

$$\begin{array}{rcrcr} 300 & \div & 3 & = & 100 \\ 30 & \div & 3 & = & 10 \\ 6 & \div & 3 & = & 2 \\ \hline \end{array}$$

[*Ans.* 112

QUESTIONS. 1. What is Division? 2. How many numbers are required to perform the operation? 3. and what are they called? 4. How is the dividend to be regarded? 5. What are the two factors, which, multiplied together will produce the dividend? 6. How is Division proved?

By carrying on the process partly in the mind, the operation may be made still shorter, thus:

3)336

112 *Answer.*

17. If 4 shares of bank stock cost 456 dollars, what will one share cost?

Operation.

4)456

114 *Answer.*

We first set down the number to be divided, or the dividend; at the left of this number, place the number by which we divide, or the divisor. Taking the first left hand figure, or hundreds, we find how many times the divisor is contained in it. The number of times, or 1, we place directly under the divided figure. We next divide the tens of the dividend: 4 is contained in 5 once, and 1 ten over, which also must be divided. If this ten be added to the unit, it will make 1 ten and 6 units—equal to 16 units; 4, then, is contained in 16, 4 times, which we place in the column of units; and in 456, 114 times.

18. Six brothers received a legacy of 1512 dollars. What was the share of each?

Operation.

6)1512

252 *Answer.*

In this question, the divisor is not contained in the first figure of the dividend. We, therefore, take the next figure, 5, which with the 1 makes 15 hundred; 6 is contained in 1500, 200 times, and 30 tens, or 300 over. This, added to the 1 ten in the next column, is 31 tens; 6 is contained in 31, five times, or in 31 tens, 50 times, and 1 ten over, which, with the two units, makes 12 units: this, divided by 6, is 2 units. The answer, then, is 200+50+2=252.

From the foregoing, we derive the following

RULE.

Write the divisor at the left hand of the dividend, with a line drawn between them.

Find how many times the divisor is contained in the first figure or figures of the dividend, setting the result directly

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QUESTIONS. 7. How is the process partly carried on in Short Division? 8. What is the method of procedure in the 17th question? 9. What is the rule for Short Division?

under the divided figure. The remainder, if there be any, carry to the next figure, calling it so many tens.

Find how many times the divisor is contained in this dividend, and set it down as before; and so continue to do until the figures in the dividend are all divided.

EXAMPLES.

1. Paid 150 dollars for 6 tons of hay. How much was it a ton? *Answer*, 25 dollars.

2. In a certain town there are 1280 inhabitants. The average number in each family is 8. How many families are there? *Answer*, 160.

3. How many yards of cloth can be bought for 1155 dollars, at 7 dollars per yard? *Answer*, 165 yards.

4. If a man labour one month for 12 dollars, how many months will he labour for 1008 dollars? and how many years, allowing 12 months to a year? *Answer*, 84 months—7 years.

5. If 15 horses consume 2550 bushels of oats in one year, how many will one horse consume?

It will be seen that 15 is a composite number, produced by the multiplication of 5 and 3, thus: $5\times3=15$. As Division is the reverse of Multiplication, it is evident, that when the divisor is a composite number, we may divide, first by one of the component parts, and that quotient by the other. For example: Suppose 30 apples to be divided equally between 15 boys. In the first place, we divide the whole number by 5. If there were only 5 boys, they would receive 6 apples each; but, as there are 3 times 5 boys, they can have only one third as many as 5 boys would have.

Operation.

5)2550

3)510

170 *Ans.*

6. How many days would it take a man to travel from Boston to New-York, travelling at the rate of 30 miles a day, the distance being 240 miles? *Answer*, 8 days.

7. If a horse travel 2184 miles in 42 days, how many miles will he travel in one day? *Answer*, 52 miles.

8. If 112 barrels of flour cost 672 dollars, what will 1 barrel cost? *Answer*, 6 dollars.

QUESTIONS. 10. When the divisor is a composite number, how may you proceed? 11. What is the first step in the 5th example? Second step?

LONG DIVISION.

1. If 2624 bushels of corn be divided equally among 41 men, how many bushels will each receive?

As 41 is greater than 12, and not a composite number, the operation must be performed by the whole divisor at once. To carry on the computation in the mind would be difficult; we must, therefore, institute a different mode from that in which the preceding questions have been performed.

Operation.

```
41)2624(64
   246
   ———
    164
    164
    ———
```

Setting down the numbers as before directed, and taking 41 for the divisor, we find that it is not contained in the 1st figure, nor in the 1st and 2d taken together, but it is contained in the first three figures, 6 times; that is, 41 is contained in 262 tens, 60 times and something over. To find what this remainder is, we find the product of 6 times 41, which we place under the three figures employed in the dividend, and subtracting it therefrom, we find the remainder to be 16, which is 16 tens. We next bring down the 4 units of the dividend, and place them at the right hand of the 16 tens, which make 164 to be divided by 41, which is contained in it 4 times. The answer, then, is 64.

By examining the work of the last question, it will be seen, that it is the same as Short Division, only that the operation is all set down, instead of being carried on in the mind. For example—divide 868 by 7, Long Division.

Operation.

```
7)868(124
  7
  ——
  16
  14
  ——
   28
   28
   ——
```

By Short Division, we say, 7 in 8 once, and 1 over; 7 in 16, twice, and 2 over; 7 in 28, 4 times, and no remainder.

In Long Division, we say—7 in 8 once, and place 1 for the first figure in the quotient; multiply 7 by this quotient

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QUESTIONS. 12. What is the difference between Long and Short Division?

figure, and place the result under the 8, and subtracting it we find the difference to be 1, to which we bring down the next figure for a new dividend, and proceed as before.

From the preceding explanations, the following RULE is deduced.

Place the divisor at the left hand of the dividend. Draw a line at the right and left of the dividend, and find how many times the divisor is contained in as many figures as will contain it once or more. Place the number of times at the right hand of the dividend, for the first figure of the quotient. Multiply the divisor by this quotient figure, and place the result under the divided figures; find the difference between them, by subtraction, and to this difference bring down the next figure of the dividend, and divide as before; so continue to do until all the figures of the dividend are brought down. Should it be necessary to bring down more than one figure to contain the divisor, a cipher must be added to the quotient.

EXAMPLES.

1. A man raised 6996 bushels of potatoes on 33 acres. How many did he raise per acre? *Answer*, 212 bushels.

2. How many years in 32485 days, if 365 days make a year? *Answer*, 89 years.

3. A legacy of 15808 dollars was left to a certain number of men, giving them 832 dollars each. How many men were there? *Answer*, 19 men.

4. How many pounds in 11520 farthings, there being 960 farthings in one pound? *Answer*, 12 pounds.

5. How many hogsheads in 49896 pints, if 504 pints make one hogshead? *Answer*, 99 hogsheads.

6. There are 8 furlongs in one mile; how many miles in 123 furlongs?

Operation.

8)123

15$\frac{3}{8}$ *Answer.*

By dividing 123 by 8, we find the quotient to be 15 miles, and there is a remainder of 3 furlongs. As it takes 8 furlongs to make a mile, it is evident that 1 furlong is $\frac{1}{8}$ of a mile, and 3 furlongs are $\frac{3}{8}$ of a mile, and 8 furlongs are $\frac{8}{8}$, equal one mile. The answer, then, is 15 miles and

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QUESTIONS. 13. Rule for Long Division?

$\frac{3}{8}$, which we place at the right hand of the quotient. We have, then, this rule, when there is a remainder in Division: Place it at the right hand of the quotient, as the numerator of a fraction, and under it place the divisor, as a denominator. A number like this is called a mixed number: Thus, 15$\frac{3}{8}$ is a mixed number. To prove this last question, we multiply the quotient by the divisor, and add to the product the numerator of the fraction, or the remainder, thus:

$$\begin{array}{r} 15\frac{3}{8} \\ 8 \\ \hline 123 \end{array}$$

7. What is the quotient of 1832 divided by 16? *Answer*, 114$\frac{8}{16}$.

8. There are 320 rods in a mile. How many miles in 66327 rods? *Answer*, 207$\frac{87}{320}$.

9. How many miles from Boston to Providence, the distance being 12800 rods? *Answer*, 40 miles.

10. If 10 shares in a factory be worth 2220 dollars, what is 1 share worth? *Answer*, 222 dollars.

To divide by 10, 100, 1000, or any number of ciphers, cut off as many figures from the right hand of the dividend as there are ciphers at the right hand of the divisor. The figures cut off on the left of the dividend, will be the quotient, and those on the right hand will be the remainder.

Operation.
1|0)222|0

11. Divide 18986421 by 10000.

Operation.
1|0000)1898|6421

Answer, 1898,6421

12. Divide 342871 by 7000.

Operation.
7|000)342|871

Answer, 48,6871

In this example, cut off the ciphers and figures of the dividend, as before, and divide by 7; to the right hand of the remainder bring down the figures which are cut off from the dividend.

13. What is the quotient of 421998 divided by 8400?

QUESTIONS. 14. When there is a remainder, after dividing, what is to be done with it? 15. What is a number like 15 3-8 called? 16. When there are ciphers at the right hand of the divisor, how may you proceed? 17. How must the figures cut off from the right hand of the dividend, be placed?

SUPPLEMENT TO
MULTIPLICATION AND DIVISION.

MULTIPLICATION.	DIVISION.
1. What will 1574 yards of cloth cost, at 12 dollars per yard?	2. If 1574 yards of cloth cost 18888 dollars, what will 1 yard cost?
3. How many inches in 56541 feet?	4. How many feet in 678492 inches, if 12 inches make 1 foot?
5. If a man travel 38 miles in a day, how many will he travel in 16 days?	6. If a man travel 608 miles in 16 days, how many will he travel in one day?
7. If 60 minutes make 1 hour, how many minutes in 13070026 hours?	8. How many hours in 784201560 minutes?
9. How many hours in 336 days?	10. In 8064 hours, how many days?
11. The quotient of two numbers is 46, and the divisor 14; what is the dividend?	12. The product of two numbers is 644, the multiplier 14; what is the multiplicand?
13. The quotient of two numbers is 72, the divisor 84; what is the dividend?	14. The product of two numbers is 6048, the multiplicand 84; what is the multiplier?
15. How many pounds of flour may be put into 640 barrels, containing each 196 pounds?	16. A man has 125440 pounds of flour, to be put into barrels containing 196 each. How many barrels must he have?
17. What will 24 oxen cost at 46 dollars each?	18. If 24 oxen cost 1104 dollars, what do they cost apiece?
19. A carriage wheel that will turn 340 times in a mile, how many times will it turn in going from Boston to New-York, it being 240 miles?	20. If a carriage wheel turn round 81600 times between Boston and New-York, and turn 340 times in a mile, what is the distance?
21. If 33 men do a piece of work in 24 days, in what time will one man do it?	22. If 1 man do a piece of work in 792 days, in what time will 33 men do it?
23. In 7894 feet, how many barley-corns?	24. How many feet in 284184 barley-corns?

MULTIPLICATION AND DIVISION,

BY CANCELLING.

The operation of questions, involving Multiplication and Division, may be greatly abridged by the following

RULE.

I. Draw a perpendicular line, and place the numbers to be multiplied on the right, and the numbers by which you are to divide, on the left hand.

II. If there be two equal numbers on each side of the line, cross them out, and omit them in the operation.

Thus: Multiply 8 by 9, and divide by 8. As 8 is found on both sides of the line, cross them both, and 9 remaining on the right, is the answer.

Operation.

$$\begin{array}{r|l} & \cancel{8} \\ \cancel{8} & 9 \\ \hline & 9 \end{array}$$ *Ans.*

III. If a number on one side of the line will divide a number on the other side, without a remainder, erase both numbers, and substitute for the larger the number of times it contains the smaller. Multiply the remainders together, on the right, for a dividend, and the remainders on the left, for a divisor.

Thus: Multiply 6 by 3, and divide by 2. In this example, 6 on the right will contain 2 on the left 3 times. Write 3, the number of times 6 contains 2, against 6. The remainders, then, on the right, multiplied together, $3\times3=9$, the answer. As there are no remainders on the left, the divisor is 1, and the quotient of 9, divided by 1, is 9. When there is no remainder on either side of the line, and the numbers are all cancelled out, the answer is 1; that is, the right hand side contains the left hand, once.

$$\begin{array}{r|l} & \cancel{6}\ 3 \\ \cancel{2} & 3 \\ \hline & 9 \end{array}$$ *Answer.*

Multiply 8 by 5, and divide the product by 3; multiply the quotient by 18, and divide the product by 9; multiply again by 9, and divide the product by 6; multiply the quo-

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QUESTIONS. 18. What is the rule for multiplication and division by cancelling? 19. First, second and third steps? 20. Is the answer affected by striking out equals on each side of the line? 21. What is done with remainders? 22. When there is no number left on either side of the line, what is the answer?

tient by 24, and divide the product by 12; multiply the quotient by 2, and divide the product by 4.

Operation.

$$\begin{array}{r|l} & 8 \\ \cancel{3} & 5 \\ \cancel{9} & \cancel{18} \\ \cancel{6} & \cancel{9} \\ \cancel{12} & \cancel{24} \\ \cancel{4} & \cancel{2} \\ \hline & 40 \text{ Answer.} \end{array}$$

Or thus:

$$\begin{array}{r|l} & 8\times5=40. \\ \cancel{3} & 5 \\ \cancel{9} & \cancel{18}=1 \\ \cancel{6} & \cancel{9} \;=1 \\ \cancel{12} & \cancel{24}=1 \\ \cancel{4} & \cancel{2} \\ \hline & 40 \text{ Answer.} \end{array}$$

Having stated the question, according to the foregoing Rule, we proceed to cancel, or cross equals on each side of the perpendicular line. In the first place, 9 is found on each side of the line. We therefore cross them both; for 9 is contained in 9 once; and multiplying any number by 1 does not alter its value. Secondly, 3 and 6, on the left hand of the line, multiplied together, make 18—equal to 18 on the right hand of the line, which may be crossed out. Again: 4 and 12, on the left, multiplied together are 48, equal to the numbers 2 and 24 on the right multiplied together, and may be crossed out. The numbers now are all cancelled, except the 5 and 8 on the right, which, multiplied together, give 40—the answer.

2. A boy gathered 16 nuts under each of 4 trees, and divided them equally between himself and 7 school mates. How many did each receive?

Operation.

$$\begin{array}{r|l} & \cancel{16}\ 2 \\ \cancel{8} & 4\times2=8. \\ \hline & 8 \text{ Answer.} \end{array}$$

NOTE. A stroke drawn through any number denotes its being cancelled; and any number which takes its place, may be set along side of it.

In the last example, it is evident, that had the boy gathered but 16 nuts, there would have been but 2 apiece; but as he gathered the same number under each tree, the 16 must be multiplied by 4; and as there were 8 to share

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QUESTIONS. 23. Give the reason for placing 16 and 4, in question 2d, on the right of the line, and 8 on the left?

them, the product of 16 multiplied by 4 must be divided by 8.

3. Multiply 20 by 5, and divide by 6; multiply by 7 and divide by 14; multiply this again by 6, and divide by 10 and multiply by 12. *Answer*, 60.

4. Multiply 120 by 40, divide by 400, multiply by 20, divide by 30, multiply by 250, divide by 50, multiply by 300, divide by 500, and give the answer. *Answer*, 24.

SUPPLEMENT

TO THE FOUR FUNDAMENTAL RULES OF ARITHMETIC, VIZ.

ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION.

EXAMPLES.

1. A man purchased a farm for 6720 dollars; sold it for 199 dollars more than he gave. For how much did he sell it? *Answer*, 6919 dollars.

2. Suppose a tree broken by the wind 39 feet from the ground, and the part broken off to be 56 feet in length, how high was the tree? *Answer*, 95 feet.

3. A merchant having 784 bushels of salt, sold 99 bushels. How many had he left? *Answer*; 685.

4. A man left his estate, valued at 8956 dollars, to his wife and daughters, giving his wife 4688 dollars. How much did the daughters receive? *Answer*, 4268 dollars.

5. Sir Isaac Newton was born in the year 1642, and died in the year 1727. What was his age? *Answer*, 85 years.

6. The greater of two numbers is 624; their difference is 89. What is the less number? *Answer*, 535.

7. What will 58 yards of broadcloth cost, at 4 dollars per yard? *Answer*, 232.

8. Bought 122 bushels of wheat, at 2 dollars a bushel; 8 oxen for 27 dollars each; 4 cows, 16 dollars each, and a waggon for 60 dollars. How much was paid for the whole, and how much more for the wheat and oxen than for the cows and waggon? *Answer*, $\left\{ \begin{array}{l} 584. \\ 336. \end{array} \right.$

9. The factors of a certain number are the difference between 1632 and 1700, and between 94 and 5 dozen. What is that number? *Answer*, 2312.

10. How many barrels of flour may be bought for 6721 dollars, at 13 dollars per barrel? *Ans.*, 517 barrels.

11. Paid 57600 cents for eggs, paying at the rate of 12 cents a dozen. How many dozen did I buy?

Answer, 4800 dozen.

12. What will 168 firkins of butter cost, at 29 dollars a firkin? *Answer*, 4872 dollars.

13. A man bought at vendue the following articles, viz. A colt for 18 dollars; a horse for 4 times as much as the colt; a waggon for 8 dollars less than the cost of the horse; 4 cows for 4 dollars more than the cost of the waggon; 12 sheep, at 3 dollars each; a plough for 5 dollars; a ton of hay for 16 dollars, and a pair of oxen for four times the cost of the hay. Now, supposing he sells the whole for 527 dollars, how much does he gain; and, if with the gain he pays 4 men, to whom he is in debt, equal sums, what does each receive? *Answer*, 46 dollars.

14. How many square feet in a board 12 feet long, and 2 feet wide?

It is evident, that a board 12 feet long and 1 foot wide would contain 12 square feet; then a board of the same length and 2 feet in width would contain twice as many feet. The answer, then, is $12\times2=24$ feet.

15. How many feet in length is a board which contains 24 square feet, and is 2 feet in width? *Ans.*, 12 feet.

It is evident, that this question is the reverse of the preceding. Then, $24\div2=12$.

16. How many square feet of boards in a log which will make 26 boards, 15 feet in length, and 3 feet in width?

Answer, 1170.

17. How many square feet will it take for the floor of a hall, 40 feet long, 22 in width, allowing 24 feet for waste?

Answer, 856 feet.

18. What is the width of a house which is 42 feet long, and the length and width multiplied make 1260 feet.

Answer, 30 feet.

19. Supposing it take 60 yards of carpeting to cover the floor of a room 15 feet in width, what is the length of the room, and how much will be the cost of the carpeting at 1 dollar 50 cents per yard? *Ans.* { 12 yds. in length. 90 dollars.

20. How much money will a man lay up in a year of 52 weeks, if he lay up 25 cents a day, Sundays excepted.

Answer, 7800 cents.

21. What is the difference between 7 times 35, and 7 times 5 and 30? *Answer*, 180.

22. How many days, months and years will a man be in travelling around the globe, it being 25000 miles, at the rate of 5 miles per hour, 10 hours in a day?

23. The less of two numbers is 432—the difference between them is 175. What is the greater? *Ans.*, 607.

24. The remainder of a sum in Division is 423; the quotient 423; the divisor is the sum of both and 19 more. What, then, was the number to be divided?

Answer, 366318.

25. What number multiplied by 72084 will produce 5190048? *Answer*, 72.

26. The remainder of a sum in division is 244; the quotient 1269; the divisor is twice the sum of the remainder, less 32. What was the sum divided?

Answer, 578908.

27. What is that number, which being divided by 7, the quotient resulting multiplied by 3, that product divided by 5, from the quotient subtract 20, to the remainder add 30, and half the sum shall make 35? *Ans.* 700.

FRACTIONS.

THUS far our operations have been confined to whole numbers, and the method of expressing them has been shown in Notation; but in business affairs it often becomes necessary to take into account something less than a whole number, or a unit. We have seen that the characters, 1, 2, when written alone express their own value; that is, 1 unit, 2 units; that, when taken together, they express either 12 or 21. To express one half of a unit, or 1, we make use of the same figures, thus: $\frac{1}{2}$. The unit is here divided into two parts, and one of those parts is here expressed. We may divide the unit, or 1, into 2, 3, 4, 5, 6, 7, 8, or 9 parts, and express the number of parts taken by writing one of the nine digits over that figure which divides the unit. *Example:* John divides an apple into four parts. He retains one part, and gives his brother three parts, which would be expressed thus: $\frac{1}{4}$, $\frac{3}{4}$.

The figure below the line shows into how many parts the apple is divided, and is called the *denominator.* The figure above the line, shows how many parts are contained in the fraction, and is called the *numerator.*

From the above, we derive the following definition:

A *fraction* is a part of any thing. *It is derived from the Latin word* frango, *which signifies* to break. When, therefore, any thing is broken into parts, those parts are called *fractions.* If a stick be broken into pieces, each piece becomes a fraction of the whole. Fractions are of two kinds, *decimal,* and *vulgar,* or common. Vulgar, when the unit is divided by any of the nine digits; that is, a *vulgar fraction* may express any assignable part of a unit, thus: $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$, &c., are vulgar fractions. But when the unit is divided by 10, or into parts expressed by 1 with any number of ciphers, as 10, 100, 1000, &c., it is called a *decimal fraction.*

DECIMAL FRACTIONS.

The term *decimal* signifies tenths. *It is derived from the Latin word* decem, *which signifies* ten. It is, therefore, applied to all fractions whose denominator is 10, or 1, with any number of ciphers. If a dollar be divided into ten parts, one of these parts, being worth ten cents, is one tenth of a dollar. If the dollar be divided into one hundred parts, one of these parts is the one hundredth part of a dollar. It is, nevertheless, a decimal fraction, because 100 is the product of 10's. The same may be said of a thousand, or ten thousand. A fraction is always known to be decimal, if its denominator be ten, a hundred, or a thousand. The denominator of a decimal fraction is not always expressed, but it can always be ascertained by the numerator. If it contains but one figure, the denominator is ten; if two, it is a hundred, &c. It is always 1, with as many ciphers annexed as the numerator has places.

When the denominator is not expressed, the fraction is distinguished from a whole number by a comma placed at the left of it. [The comma is called the separatrix.] *Example:* ,5 ,50 is read five tenths, fifty hundredths, as though they were written $\frac{5}{10}$, $\frac{50}{100}$. If the numerator have not so many places as the denominator has ciphers, supply the defect by prefixing ciphers, thus: for $\frac{5}{100}$, write ,05; $\frac{5}{1000}$ write ,005. Ciphers placed at the right hand of a decimal, do not affect its value, as $\frac{5}{10}$, $\frac{50}{100}$ are the same in value; for while the addition of the cipher indicates a division into parts ten times smaller than the preceding, it makes the decimal express ten times as many parts. Thus, 5 tenths denotes 5 parts of a unit, which is divided into 10 parts; and 50 hundredths denotes 50 parts of a unit, which is divided into 100 parts. It is, therefore, plain, that the value is not altered, since 5 is half of 10, and 50 is half of 100.

The value of a decimal depends upon its distance from the unit's place; as whole numbers increase from the unit's place towards the left in a tenfold proportion, so decimals, in the same ratio, decrease from the unit's place towards the right hand; as will appear from the following

TABLE.

Millions.	Hundreds of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.	Tenth parts.	Hundredth parts.	Thousandth parts.	Ten thousandth parts.	Hundred thousandth parts.	Millionth parts.
7	6	5	4	3	2	1	2	3	4	5	6	7

From the above Table it is evident that each figure, whether a whole number or decimal, takes its value from the unit's place. If it be in the first place after units, it is tenths; if in the second, it is hundredths, &c. Consequently, every decimal will have for its denominator 1, with as many ciphers as the decimal is distant from the unit's place; thus—2 in the Table is $\frac{2}{10}$; 3 is $\frac{3}{100}$; 4 is $\frac{4}{1000}$, &c.

Although ciphers placed at the right hand of a decimal fraction do not affect its value, yet, placed at the left, they

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QUESTIONS. On what does the value of a decimal depend? In what proportion do decimals decrease from the units' place towards the right? From what does each figure in the table take its value? What is the value of the first figure at the right?

diminish it in a tenfold proportion, by removing the significant figure so much farther from the unit's place. As—,5 ,05 ,005 express different values, viz.—,5 is $\frac{5}{10}$, ,05 is $\frac{05}{100}$, ,005 is $\frac{005}{1000}$. As whole numbers are written units under units, tens under tens, from right to left, so decimals are written tenths under tenths, from left to right.

Example 1. Write 2 tenths; 3 hundredths; 4 thousandths; 6 ten thousandths.

,2
,03
,004
,0006

2. Write twenty-nine thousandths; three hundred and fourteen thousandths; five ten thousandths, and sixty-seven millionths.

,029
,314
,0005
,000067

3. Write five tenths; five hundredths; fifty thousandths, and forty-nine; one hundred thousandths, and sixteen thousandths.

ADDITION OF DECIMALS.

RULE.

Place the numbers so that the decimal points may stand directly under each other. Add as in whole numbers; observing to point off as many places for decimals in the amount, as will be equal to the greatest number of decimals in any of the given numbers.

EXAMPLES.

1.	2.	3.
		2345,6
Add 459,51	79,01	1987,51
371,62	891,67	3456,712
129,03	137,79	21098,6543
1271,007	1239,812	16723,24567
1090,215	2671,927	65431,002001
3321,382		

QUESTIONS. What is the rule for addition of decimals?

4. Add thirty-five and four tenths; five hundred twenty-nine and seven millionths; sixty-nine, four hundred and sixty-three thousandths; two hundred, sixteen and two hundredths; seventy-seven, nine hundred and two tenths.

Answer, 1827,083007.

5. Add forty-nine and sixty-seven hundredths; six hundred seventy-nine, two hundred seventy-five thousandths; one thousand four hundred, fifty-five thousandths, nine hundred and ninety-nine millionths.

6. Add 249,39—6712,9123—6,3219—2739,235—5,671—723,2674—926,679—72,601.

7. Add ,7+9, 2 +, 321+279,+4, 67+349, 2+3, 956.

8. Purchased of one man 325,5 lbs. of beef—of another, 175,75—of another 178,028; what was the amount?

9. I receive of A $183,25—of B $138,89—of C $372,218—of D $88,99—of E $139,29; what is the amount of the whole?

10. Add $59,67—$158,355—$375,752—$167,375—$567,756

SUBTRACTION OF DECIMALS.

RULE.

Write the numbers and point the result, as in Addition of Decimals, and subtract as in whole numbers.

	1.	2.	3.	4.
From	429,67	87,02	359,76	2029,5
Take	319,76	59,2	126,571	1718,279
	109,91			

5. From two hundred and sixty-nine and three tenths, take fifty-seven and thirty-nine hundredths.

Answer, 211,91.

6. Take twenty-four thousandths from nine hundredths.

Ans. ,066.

7. Take sixty-five millionths from five tenths.

8. From three hundred seventy-five thousand and three tenths, take two hundred forty-nine and thirty-nine one hundred thousandths. *Ans.* 374751,29961.

9. From 361,2 take 276,75.

QUESTIONS. Rule for the subtraction of decimals?

10. From 456,35 Take 27,356.
11. From 5678,0002 Take 3980,96715.

MULTIPLICATION OF DECIMALS.

RULE.

Multiply as in whole numbers, and point off as many places for decimals in the product as there are decimal places in both factors.

If there are not so many places, supply the defect by prefixing ciphers.

EXAMPLES.

1. Multiply 49,5 by 3,2. 2. Multiply 569,39 by 27,05.

```
  3,2
 ─────
  99,0
 1485,
 ─────
 158,40
```

3. Multiply 6,791 by 2,67.
4. Multiply 549,05 by 35,257.
5. Multiply six hundred and seventy-five by twenty-seven and three tenths.
6. Multiply sixty-seven thousand by three hundredths.
7. Multiply 34,56 by 1,3. 8. Multiply 674,49 by 37,16.
9. Multiply 5648 by 6,78.
10. Multiply 7864 by 467.
11. Multiply fifty-seven and three tenths by twenty-nine.
12. Multiply thirty-seven thousand by three hundredths.
13. Multiply fifty thousand and seven tenths by four hundredths.
14. Multiply sixty-nine and five tenths by three thousandths.

DIVISION OF DECIMALS.

RULE.

Divide as in whole numbers, and point off so many places in the quotient for decimals as the decimal places in the dividend exceed those of the divisor; or, so many that

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QUESTIONS. What is the rule for the multiplication of decimals? Rule for the division of decimals?

the decimal places in the quotient and divisor taken together shall equal the decimal places in the dividend.

EXAMPLES.

1. Divide 36,72 by 18.

```
18)36,72(2,04
   36
   ---
     72
     72
```

2. Divide 21,7 by 7.

```
7)21,7
  ----
   31,
```

3. Divide 2,17 by 7. *Answer*, ,31.

4. Divide ,217 by ,7. *Answer*, ,31.

5. Divide ,217 by 7. *Answer*, ,031.

6. Divide one hundred and seventeen and nine tenths by nine tenths. *Answer*, 13,1.

7. Divide four hundred fifty-six and three hundred thirty-three thousandths by three hundredths.

8. If three hundred fifty pounds of beef cost twelve dollars twenty-five hundredths, what cost one pound? *Answer*, ,035.

9. If 565,05 pounds cost 25,42725 dollars, what will one pound cost? *Ans.* ,045.

FEDERAL MONEY.

FEDERAL MONEY is the coin of the United States; its denominations are eagles, dollars, dimes, cents and mills.

From the above examples and illustrations in decimal fractions, we have seen that a decimal is the division of the unit into tens, and that from the unit's place towards the right hand it decreases in a tenfold proportion. If we examine the denominations of Federal Money, we shall find that all bear a decimal relation to the dollar, which is considered the unit. This will be seen by the following

TABLE.

10 Mills =1 Cent.
10 Cents =1 Dime.
10 Dimes =1 Dollar.
10 Dollars=1 Eagle.

NOTE. The eagle is a gold coin, the dollar and dime are silver coins, the cent is a copper coin. The mill is only imaginary, there being no coin of that denomination.

QUESTIONS. What is Federal Money? Its denominations?

The dime being 1 tenth of a dollar, it occupies the first, or right hand place from the dollar—thus, 0,1. The cent, being 1 tenth of a dime, and consequently 1 hundredth of a dollar, occupies the second place, or place of hundredths —thus, 0,01. The mill, being 1 tenth of a cent, and consequently 1 thousandth of a dollar, occupies the third place, or place of thousandths—thus, 0,001. Placing them
d. d. c. m.
together, 1 1 1 1. This may be read one dollar, one dime, one cent and one mill; or, one dollar, eleven cents and one mill—as eleven cents is equal to one dime and one cent. The same may be said of eagles and dollars—thus, 25 dollars may be read 2 eagles and 5 dollars, since 20 dollars are equal to 2 eagles. Write 4 eagles, 5 dollars, 8
E. D. D. C. M.
dimes, 3 cents, 5 mills—4 5 8 3 5. This may be read 4 eagles, 5 dollars, 8 dimes, 3 cents and 5 mills; or, 45 dollars, 83 cents and 5 mills. Hence, it is evident that the denominations in Federal Money are dollars and decimals of a dollar, and may be treated as Decimal Fractions. Federal Money is denoted by this character $ placed before the figure.

ADDITION OF FEDERAL MONEY.

RULE.

Write the denominations, and point the result as in Addition of Decimals.

EXAMPLES.

1. If I buy a bushel of wheat for $2,25—a bushel of corn for $1,32—four yards of cloth for $14,28,5; how much do I pay for the whole?

2,25
1,32
14,28,5

$17,85,5 *Ans.*

2. Bought 8 yards of cloth for $16,25½ cents—a pair of shoes for 87½ cents—a hat for $4,33—a whip for ,42 cents —a knife for ,37½ cents; how much did I pay for the whole? *Ans.* $22,25,5.

3. Bought a cart for $17,62—a wagon $62½—a plough

$7,48 cents—4 rakes $1,26—3 hoes $2,15—a pitch-fork ,87 cents; how much did the whole cost? *Ans.* $91,88.

4. Purchased a barrel of flour for $9,25—4 pounds of tea $2,08—2 gallons of molasses ,64 cents—3 pounds of raisins ,37½ cents—9 pounds of sugar $1,22½ cents—8 yards of calico $2,23½ cents; what is the amount of the whole? *Ans.* $15,80,5.

5. Add forty dollars, sixty-seven cents and three mills; six hundred seventy-nine dollars, twenty-five cents and seven mills; one thousand and four dollars, five cents and five mills; nine hundred ninety-nine dollars, thirty-nine cents and nine mills? *Ans.* $2723,38,4.

SUBTRACTION OF FEDERAL MONEY.

RULE.

Write the numbers, and point the result as in Subtraction of Decimals.

EXAMPLES.

1. A man bought 50 bushels of wheat for $125,50—sold it for $145,75; how much did he gain? *Ans.* $20,25.
2. Bought 26 bushels of oats for $8,49—sold the same for $8,94; how much did I gain? *Ans.* $0,45.
3. Purchased a horse for $92—lost on the sale of him $15,25; for how much did I sell him? *Ans.* $76,75.
4. Bought 2 barrels of flour for $22,50—but, it being damaged, I am willing to sell it at $4,25 less; what must I receive for it? *Ans.* $18,25.
5. Bought 8 yards of cloth for $36—gave a $50 bill; what must I receive in change? *Ans.* $14.
6. Subtract 1 mill from $333. *Ans.* $332,99,9.
7. Subtract half of a cent from $100,000.
8. Bought a wood lot for $879—sold the same for $1000,81; how much did I gain?
9. If a man's wages in a year amount to $1434, and he spends $928,45, how much does he save at the end of the year?
10. How much must be added to $32,50 to make $1000?

MULTIPLICATION OF FEDERAL MONEY.

RULE.

Write the numbers, and point the product as in Multiplication of Decimals.

EXAMPLES.

1. How much will six pairs of shoes cost at $1,37½ a pair? *Answer*, $8,25. *Operation.*

1,37,5
6
$8,25,0

It will be seen that the operation is the same as in simple numbers. The product will always be in the lowest denomination of the given sum, until distinguished by points.

2. What will 9 sheep cost at $3,75 each? $33,75.

3. How much must be paid for 45 bushels of corn, at $1,37 per bushel?

4. What will 38 pounds of sugar cost at 13½ cts. per pound? $5,13.

5. What will 3 doz. hats cost at $4,75 each? $171.

6. What will 75 doz. eggs cost at 15½ cents a dozen? *Answer*, $11,62,5.

7. How much will a man spend in a year, if he spend 12½ cents a day?

8. What will 55 yards of broadcloth cost at $3,87½ per yard?

DIVISION OF FEDERAL MONEY.

RULE.

Write the numbers, and point the quotient as in Division of Decimals.

EXAMPLES.

1. Bought 8 bushels of wheat for $17,92. How much was it per bushel? *Answer*, $2,24.

2. Bought 9 pounds of tea for $3,37½. What was it per pound? *Operation.*

9)3,37,5.
,37,5 *Ans.*

3. Bought tea to the amount of $3,37½, at 37½ cents per pound. What quantity did I buy? *Ans.*, 9 lbs.

4. Bought 14½ bushels of corn for $21,75. How much was it per bushel? *Answer*, $1,50.

5. If a man pay $38,43,7½ for 20½ casks of lime, how much was it per cask? *Answer*, $1,87½.

6. Bought 6 yoke of oxen for $450. What was paid for each ox? *Answer*, $37,50.

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SUPPLEMENT

TO DECIMAL FRACTIONS AND FEDERAL MONEY.

1. Purchased 49,5 pounds of butter of A, at 12½ cents per pound; 37,51 pounds of B, at 18¾ cents per pound; 155,05 pounds of C, at 20 cents per pound. How many pounds did I buy, and what was the cost of the whole?

Answer, { $124,05,5,7.
242,06 pounds.

2. When butter is worth 18 cents 4 mills per pound, how many pounds can be bought for $671,60? *Answer*, 365 pounds.

3. At 9 mills per yard, how many yards of tape can be bought for 45 dollars, 81 cents, 9 mills? 5091 yards.

4. If 5091 yards of tape be worth $45,81,9, what is 1 yard worth? *Answer*, 9 mills.

5. What will 629,21 feet of boards cost at $20,18 per thousand?

6. If 40 yards of broadcloth cost $375, what cost 1 yard? *Answer*, $9,37,5.

7. What will 36 bushels 9 tenths of corn amount to, at 1 dollar 5 tenths per bushel? *Answer*, $55,35.

8. If corn be worth 3 and 5 tenths as much as potatoes, which are worth 25 hundredths of a dollar per bushel, and rye 5 tenths more than corn, and wheat 2 and 4 tenths more than rye, what is the value of wheat? $3,15.

9. Bought 4 cords of wood for $12,28; 15 pounds of beef for $1,25. How much do I pay for the whole, and how much more for the wood than for the beef?

10. How much must be paid for 36½ bushels of peas, at $2,87½ cents per bushel? *Answer*, $104,93,7½.

11. Bought 28 bushels of potatoes, at 28 cents a bushel; 45½ bushels of apples, at $1,12½ per bushel. How much

did the whole cost, and how much more did the apples cost than the potatoes?

Ans. { $59,02,7½ whole cost.
43,34,7½ difference.

It is customary for the merchant, when he delivers goods, to give also a bill of the articles, and their prices, with the amount cast up. Such bills are called *Bills of Parcels.*

Concord, May 18, 1837.

Mr. John Worthy,

Bought of Peter Trustrum,

5½ bushels of oats, at	$0,63 per bushel,	$3,46,5
12½ " wheat	1,50 " "	18,75,0
7½ cords of wood, at	3,45 per cord,	25,87,5
		$48,09,0

Received payment,

Peter Trustrum.

Mr. Benj. Savage,

Bought of Joseph Easy,

12½ yards of broadcloth,	at	$3,87½ per yard;	
5½ casks of nails,	"	5,50 per cask;	
112 pounds of iron,	"	,9½ per pound;	
16 " " steel,	"	18 " "	
25 " " lead,	"	,9¼ " "	
1 hhd. sugar, 8½ cwt.,		9,24 " cwt.;	
2½ boxes of glass,	"	7,50 per box.	
			$191,81,0

COMPOUND NUMBERS.

Numbers are called compound when they consist of different kinds or denominations, as pounds, shillings, pence and farthings; years, days, hours, &c.

TABLES OF COMPOUND NUMBERS.

MONEY.

I. *Federal Money.* [*]

10 mills make	1 cent,	*ct.*
10 cents	1 dime,	*d.*
10 dimes	1 dollar,	*dol.*
10 dollars	1 eagle	*e.*

II. *English Money.*

4 farthings, *qrs.* make	1 penny,	*d.*
12 pence	1 shilling,	*s.*
20 shillings	1 pound,	*l* or *£.*

III. *Time.*[†]

60 seconds, *s.* make	1 minute,	*m.*
60 minutes,	1 hour,	*h.*
24 hours,	1 day,	*d.*
7 days,	1 week,	*w.*
365¼ *d.* or 365,25 *d.* or 52 weeks,	1 year,	*yr.*

IV. *Troy Weight.*[‡]

24 grains, *grs.* make	1 penny weight,	*pwt.*
20 penny weights,	1 ounce,	*oz.*
12 ounces,	1 pound,	*lb.*

Apothecaries' Weight.

20 grains, *gr.* make	1 scruple,	*sc.*
3 scruples,	1 dram,	*dr.*
8 drams,	1 ounce,	*oz.*
12 ounces,	1 pound,	*lb.*

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[*] "The above denominations of Federal Money are authorized by the laws of the United States; but, in the transaction of business in New-England, we seldom hear any of them named, but dollars and cents.

"A coin is a piece of money stamped, and having legal value. The coins of the United States are three, of gold; the eagle, half eagle and quarter eagle; five of silver, the dollar, half-dollar, quarter-dollar, dime and half-dime; and two of copper, the cent and half-cent. Of the small foreign coins current in the United States, the most common are the New-England *four-pence-half-penny* or New-York sixpence, worth 6¼ cents; and the New-England *ninepence*, or New-York shilling, worth 12½ cents. The value of the several denominations of English money is different in different places. A dollar is reckoned at 4*s.* 6*d.* in England, 5*s.* in Canada, 6*s.* in New-England, Virginia and Kentucky; 8*s.* in New-York, Ohio and North-Carolina; 7*s.* 6*d.* in Pennsylvania, New-Jersey, Delaware and Maryland, and 4*s.* 8*d.* in South Carolina and Georgia."

[†] "The year is commonly divided into 12 months, as in the following table, called calendar months:

Months.	Days.	M.	D.	M.	D.	M.	D.
1 January,	31	4 April,	30	7 July,	31	10 October,	31
2 February,	28	5 May,	31	8 August,	31	11 November,	30
3 March,	31	6 June,	30	9 September,	30	12 December,	31

Another day is added to February every fourth year, making 29 days in that month, and 366 in the year. Such years are called Bissextile, or Leap Year. To know whether any year is a common or leap year, divide it by 4, if nothing remain, it is leap year; but, if 1, 2 or 3 remain, it is 1st, 2d or 3d, after leap year. The number of days in the several months may be called to mind by the following verse:

Thirty days hath September,
April, June and November;
All the rest have thirty-one,
Excepting February alone,
Which hath twenty-eight, nay more,
Hath twenty-nine one year in four.

The true solar year consists of 365 days, 5*h*. 48*m*. 57*s*., or nearly 365¼ days. A common year is 365 days, and one day is added in leap year to make up the loss of ¼ of a day in each of the preceding years. This method of reckoning was ordered by Julius Cæsar, 40 years before the birth of Christ, and is called the Julian Account, or Old Style. But, as the true year falls 11*m*. 3*s*. short of 365¼ days, the addition of a day every fourth year was too much by 44*m*. 12*s*. This amounted to one day in about 130 years. To correct this error, Pope Gregory, in 1582, ordered that 10 days should be struck out of the calendar, by calling the 5th of October the 15th; and to prevent its recurrence, he ordered that each succeeding century divisible by 4, as 16 hundred, 20 hundred and 24 hundred should be leap years, but that the centuries not divisible by 4, as 17 hundred, 18 hundred, and 19 hundred should be common years. This reckoning is called the Gregorian, or New Style. The New Style differs now 12 days from the Old Style.

‡ "The original standard of all our weights was a corn of wheat taken from the middle of the ear, and well dried. These were called grains, and 32 of them made one penny weight. But it was afterwards thought sufficient to divide this same penny weight into 24 equal parts, still calling the parts grains, and these are the basis of the table of Troy weight, by which are weighed gold silver and jewelry. Apothecaries weight is the same as Troy weight, only having different divisions between grains and ounces. Apothecaries make use of this weight in compounding their medicines, but they buy and sell their drugs by Avoirdupois weight. In buying and selling coarse and drossy articles, it became customary to allow a greater weight than that used for small and precious articles, and this custom at length established Avoirdupois, or common weight, by which all articles are weighed with the following exceptions. Avoirdupois weight is about one sixth part more than Troy weight,—the former

being 7000 grains,and of the latter 5750 grains. In buying and selling by the hundred weight, 28 pounds have been called a quarter, 112 pounds a *cwt.;* but this practice of grossing, as it is called, is now pretty generally laid aside, and 25 pounds are considered a quarter, and 4 quarters (100 pounds) a hundred weight."

Avoirdupois, or Common Weight.

16	drams	make	1 ounce,	*oz.*
16	ounces	"	1 pound	*lb.*
28	pounds	"	1 quarter,	*qr.*
4	quarters	"	1 hundred,	*cwt.*
20	hundred	"	1 ton,	*ton.*

◆

MEASURES.

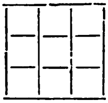
VII. *Long Measure.**

3	barley-corns	make	1 inch,	*in.*
12	inches	"	1 foot,	*ft.*
3	feet	"	1 yard,	*yd.*
5½	yards, or 16½ feet,		1 rod, or pole,	*rd.*
40	rods,	"	1 furlong,	*fur.*
8	furlongs	"	1 mile,	*mi.*
3	miles	"	1 league,	*lea.*
69½	miles	"	1 degree,	*deg.*
360	degrees	"	1 circle of the earth.	

7,92	inches	make	1 link,	*lk.*
25	links	"	1 rod,	*rd.*
4	rods, or 100 li.,		1 chain,	*cha.*
80	chains		1 mile,	*mi.*

* "The original standard of English Long Measure, was a barley-corn taken from the middle of the ear, and well dried. Three of these in length were called an inch, and then the others as in the table. Long Measure is employed for denoting the distance of places, and for measuring where length only is concerned. When measure is applied to surface where length and breadth both are concerned, it is called Square Measure. A square inch is a square measuring an inch on every side. The table of Square Measure is made from that of Long Measure, by multiplying the several numbers of the latter into themselves. Thus 12 inches are a foot in length; a square foot, then, is a square which measures 1 foot, or 12 inches on every

side, and contains $12 \times 12 = 144$ square inches. Three feet in length make a yard. A square yard is a square measuring 3 feet on each side; but such a square contains (see figure) nine ($3 \times 3 = 9$) square feet; and when we say that a surface contains so many square feet, or square yards, we mean, that the surface is equal to such a number of squares—meaning a foot, or a yard, on each side."



VIII. *Cloth Measure.*

2¼	inches	make	1 nail,	*na.*
4	nails	"	1 quarter,	*qr.*
4	quarters	"	1 yard,	*yd.*
3	quarters	"	1 ell Flemish,	E.F.
5	quarters	"	1 " English,	E.E.
6	quarters	"	1 ell French,	E.F.
37,2	in.	"	1 Ell Scotch,	E.S.

IX. *Square Measure.*

144	inches	make	1 squ. foot,	*ft.*
9	feet	"	1 squ. yard,	*yd.*
30¼	yards	"	1 squ. rod,	*rd.*
272¼	feet	"	1 squ. rod,	*rd.*
40	rods	"	1 rood,	*ro.*
4	roods	"	1 acre,	*acr.*
640	acres	"	1 squ. mile,	*mi.*

10	square chains	make	1 acre,	*acr.*
6400	chains	make	1 square mile,	*mi.*

X. *Solid, or Cubic Measure.*

1728	inches	make	1 foot,	*ft.*
27	feet	"	1 yard,	*yd.*
40	feet of round timber or		1 *ton.*	
50	feet of hewn timber			
128	feet	make	1 cord,	*cor.*

XI. *Wine Measure.**

4	gills	make	1 pint,	*pt.*
2	pints	"	1 quart,	*qt.*
4	quarts	"	1 gallon,	*gal.*
31½	gallons	"	1 barrel,	*bar.*
2	barrels	"	1 hogshead,	*hhd.*
2	hogsheads		1 pipe,	*p.*
2	pipes		1 tun,	*t.*

* "Four pounds of Troy Weight of wheat, gathered from the middle of the ear, and well dried, were called one gallon; and this was the original standard of all English measures, both liquid and dry; and this was the same as the present wine gallon. But, in time it became customary to use a larger measure in selling cheap liquors; and this custom at length established the beer measure, which bears about the same proportion to Wine Measure that Avoirdupois does to Troy Weight. The Dry Measure was also made larger than the Wine Measure, and was at length established at about a mean between Wine and Beer Measure. By Wine Measure, are measured wine, all kinds of spirits, cider, vinegar, oil, &c. By Beer Measure are measured ale and beer, and by Dry Measure all kinds of dry goods—corn, grain, salt, roots, fruit, &c. A standard bushel is 18⅛ inches diameter, and 8 inches deep. The statute bushel for measuring coal, ashes and lime, in Vermont, contains 38 quarts, or 2553.6 cubic inches."

When measure is applied to solids which have length, breadth and thickness, it is called solid or, cubic measure. A solid inch is a body or block, having six sides, each of which is an inch square, and the number of inches in a solid foot is equal to the number of such blocks that would be required to make a pile a foot square and a foot high. Now it would require 144 blocks to cover a square foot one inch high. Hence to raise the pile 12 inches high would require 12 times 144=1728 blocks, or inches. In like manner it would require 9 solid blocks, a foot each way, to cover a square yard to the height of one foot, and 3 times 9=27, to raise it 3 feet, or make one solid yard. A cord of wood is sometimes called 8 feet. In this case four feet in length, four in breadth, and six inches in height, a foot, that is, $\frac{1}{8}$ of a cord is called one foot, $\frac{2}{8}$, 2 feet, &c. In measuring lands, roads, &c. the distances are usually taken in chains and links. In ordinary business, feet and inches are the most common measures. By forty feet of round timber, in the table of solid measure, is meant so much round timber as will make forty feet after it is squared.

XII. *Beer Measure.*			XIII. *Dry Measure.*		
2 pints, *pts.* make	1 quart,	*qt.*	2 pints, *pts.* make	1 quart,	*qt.*
4 quarts,	1 gallon,	*gal.*	4 quarts,	1 gallon,	*gal.*
36 gallons,	1 barrel,	*bar.*	8 quarts,	1 peck,	*pk.*
54 gallons,	1 hogshead,	*hhd*	4 pecks,	1 bushel,	*bu.*
282 cubic inches	1 gallon,	*gal.*	8 bushels,	1 quarter,	*qr.*
			4 quarters,	1 chaldron,	*ch.*

XIV. *Circular Measure.**

60 seconds, ″ make	1 minute, ′
60 minutes,	1 degree,°
30 degrees,	1 sign, *s.*
12 signs, or 360°	1 circle.

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* "Every circle, without regard to its size, is supposed to be divided into 360 equal parts, called degrees, and these again to be subdivided into minutes and seconds; so that the absolute quantity expressed by any of these denominations must always depend upon the size of the circle. In this measure, are reckoned latitude, longitude, the planetary motions," &c. *Thompson.*

XV. *Miscellaneous.*

12 things make	1 dozen, *doz.*
12 dozen	1 gross, *gro.*
12 gross, or 144 *doz.*	1 great gross, *G. gro.*
20 things	1 score,
24 sheets of paper,	1 quire,
20 quires	1 ream,
112 pounds	1 quintal.
6 points	1 line.
12 lines	1 inch.
4 inches	1 hand.
6 feet	1 fathom.

Books.

When a sheet is folded into two leaves it is called Folio.

When folded into 4 leaves it is called Quarto.

When folded into 8 leaves it is called Octavo.

When folded into 12 leaves it is called Duodecimo, or 12*mo.*

When folded into 18 leaves it is called 18*mo.*

When folded into 24, it is called 24*mo.*

REDUCTION.

Reduction is the changing of numbers from one denomination to another, without altering their value.

1. In £6, 7s. 8d. 2qr., how many farthings?

Operation.

£6, 7s. 8d. 2qr.
20
———
127
12
———
1532
4
———
6130 qr. *Ans.*

It is plain, that, if in one pound there are 20s. in six pounds there are 6 times as many, or 120s.; and in £6, 7s, there are 127s. Again, if, in one shilling there are 12d. in 127s. there are 127 times 12, which with the 8d. added, equals 1532d. Lastly, it is evident, that, if in 1 penny there are 4 farthings, in 1532d. there are 4 times the number of farthings, or 6130, the 2qrs. in the given question being added. Hence it appears, that in any given number of pounds there are 20 times as many shillings as pounds, 12 times as many pence as shillings, and 4 times

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Questions. 1. What is Reduction? 2. By what numbers do you multiply in question 1st? and why?

as many farthings as pence. This process is called Reduction Descending, because higher denominations are brought into lower.

2. In 6130 farthings, how many pounds?

Operation.

```
 4)6130
   ----
12)1532  2qrs.
   ----
2|0)12|7  8d.
   ----
```

Ans. £6 7s. 8d. 2qrs.

It will be seen that this question is the reverse of the former; and as 4 farthings are equal to 1 penny, so the number of pence in 6130qrs. will equal the number of times it contains 4, or 1532d. and 2qrs. over. Again,—as 12d. are equal to 1 shilling, so in 1532d. the number of shillings will equal the number of times it contains 12, or 127s. and 8d. over. Lastly, since 20s. is equal to one pound, 127s. must equal 6 pounds and 7s., because 20 is contained in 127s. 6 times, and 7s. over. It is always to be remembered, that the remainder is of the same denomination as the dividend, whatever may be the divisor. This latter process is called Reduction Ascending, because lower denominations are brought into higher. By these examples, it appears that Reduction Ascending and Descending mutually prove each other. From the preceding operations we derive the following RULES:

Reduction Descending.

RULE.

Multiply the highest denomination given by as many of the next lower, as it takes to make one of the higher, observing to add the next lower denomination to the product. Multiply pounds by what makes a pound; shillings by what makes a shilling, and so on, until you have reduced it to the denomination sought in the question.

Reduction Ascending.

RULE.

Divide the denomination given by as many of that denomination as it takes to make one of the next higher. Divide farthings by as many farthings as it takes to make a penny; pence by as many pence as it takes to make a shilling, and so on, until you have reduced it to the denomination required.

QUESTIONS. 3. What is the process called? 4. By what numbers do you divide in question 2? and why? 5. What is this process called? 6. What is the rule for Reduction Ascending? 7. For Reduction Descending?

English Money.

EXAMPLES.

3. In 624 pounds, how many farthings?

5. If 6 shillings make a dollar, how many dollars in 780£ 18s?

7. How many pence in 24£ 16s 11d?

9. How many guineas, 28s each, in 49£?

EXAMPLES.

4. In 599040 farthings, how many pounds?

6. In 2603 dollars, how many pounds and shillings?

8. How many pounds in 5963 pence?

10. How many pounds in 35 guineas?

Troy Weight.

11. In 18lbs. 11oz. 6pwt. 18grs., how many grains?

13. Bought jewelry weighing 1lb. 10oz. 15pwt. 20grs. paid $0,04 per grain. What did I pay?

12. In 109122 grains, how many pounds?

14. If I pay $437,60 for jewelry, at the rate of $0,04 per grain, what was the weight?

Avoirdupois Weight.

15. In 5 tons, 16cwt. 3qrs. 12lbs. 14oz. 10drs., how many drams?

17. What will be the cost of 40 tons of lead, at $0,12 per pound?

19. What must I pay for 20 tons, 18 cwt. of hay, at the rate of half a cent per pound?

16. Reduce 3350762drs. to tons.

18. Bought lead to the amount of $10752, at $0,12 per pound, what was the weight?

20. How much hay can be bought for $234,08 at half a cent per pound?

Apothecaries' Weight.

21. Reduce 6lbs. 10oz. 7drs. 2sc. 16grs. to grains.

23. Reduce 21lbs. 11oz. 3drs. 1sc. 13grs. to grains.

22. In 39836 grains, how many pounds?

24. In 126453 grains, how many pounds?

Long Measure.

25. Reduce 640 degrees to feet.

NOTE. To reduce degrees to statute miles, multiply first by 69, and for the ½ in the

26. In 234854400 feet how many degrees?

NOTE. To divide by 16½ first reduce 16½ to halves and the dividend also to multiplier take ½ of the multiplicand—Thus:

```
2)640 degrees
   69½
  ----
  5760
 3840
   320
  ----
 44480 miles.
```

27. How many barley corns will reach across the Indian Ocean, it being 45 degrees?

29. A teamster, after travelling 20 miles, met a man who offered him as many 5 cent pieces for his load, as his larger wheel had turned round since he commenced his journey, the wheel being 20 feet in circumference: how much did he get for his load?

31. How many barley corns will reach round the globe, it being 360 degrees?

halves—Thus, 16½=33 half feet; & 234854400=469708800 half feet, which divided by 33 gives 142338 rods.

28. How many degrees in 594475200 barley corns?

30. If a teamster receive for his load $264, being paid 5 cents for each revolution of the larger waggon wheel, the circumference being 20 feet, how far had he travelled?

32. In 4755801600 barley corns how many degrees?

Land, or Square Measure.

33. If in a county there are 1200 square miles, how many square rods are there in this county?

35. How many square inches in 430 square acres?

34. How many square miles in 122880000 square rods?

36. In 2697235200 square inches, how many square acres?

Solid, or Cubic Measure.

37. How many solid inches in 10 cords of wood?

39. In 40 cords, how many cord feet?

41. How many solid inches in a pile of timber containing 54 tons?

38. Reduce 2211840 solid inches to cords.

40. In 320 cord feet, how many cords?

42. In 3732480 solid inches of round timber, how many tons?

Cloth Measure.

43. Reduce 6324yds. 3qrs. 3na. to nails.

44. How many yards in 101199 nails?

45. What will 54 yds. 3qrs. 0 na. of cloth cost, at 12½cts. an inch?

46. If I buy cloth to the amount of $246 37½, at the rate of 12½ cts per inch, how many yards do I buy?

Wine Measure.

47. In 5 tuns, how many quarts?

48. In 5040 quarts, how many tuns?

49. What will be the cost of 8 hhds. of wine, at $0,05 per pint?

50. How much wine can be bought for $201,60, at 5 cents per pint?

Ale, or Beer Measure.

51. What will 40 hhds. of beer cost, at $0,02 per pint?

52. How much beer can be bought for $345,60, at 2 cents per pint?

Dry Measure.

53. Reduce 8 ch. 10 bush. 3 pks. 7 qts. 1 pt. to pints.

54. In 19135 pints, how many chaldrons?

55. How many pints in 30 bushels?

56. In 1920 pints, how many bushels?

Circular Measure.

57. In 6 s. 28 deg. 10 m. 8 sec. how many seconds?

58. Reduce 749408 seconds to signs.

59. Reduce 7 s. 21° 30′ 29″ to seconds.

60. In 833429 seconds, how many signs?

Time.

61. How many minutes in 10 yrs. 30 days, 18 hrs. 45m.?

62. In 5303925 minutes, how many years?

63. How many seconds from May 1st, 1830, to April 12th, 1837?

64. In 219326400 seconds, how many years?

65. Alexander the Great ascended the throne 332 yrs. before the Christian era.—How many minutes from that period to 1837?

66. In 1140807240 minutes, how many years?

SUPPLEMENT TO REDUCTION.

1. How many dollars in £480, 18s.? *Ans.* $1603.

2. In £332, 16s. 8d. how many ninepences? *Answer*, 8875 and 5d.

3. How many times will a regular clock strike in 400 years? *Answer*, 22776000.

4. A man sold four trees standing in the forest, measuring as follows: 6 tons, 5 tons, 4½ tons, 3 tons; at 12½ cts. per foot. What was their value? *Answer*, $92,50.

5. A man buys 20 tons of hay at 45 cts. per cwt. He pays a man 62½ cts. a day for himself, and 50 cts. for his team. It takes him 6 days to cart it. How much does the hay cost him? *Answer*, $186,75.

6. How many plank one foot wide will it take to cover a bridge 60 rods in length, and 2 rods wide, and what will it cost at 20 cents per hund. feet? *Ans.* { 990 plank. $65,34 cost.

7. If a boy be paid for wheeling a bushel of apples over said bridge 1 mill for every revolution of the wheel, which is 5 feet in circumference, how much does he receive? *Answer*, 19 cts. 8m.

8. What will 2 tons of molasses amount to at 6¼ cents a pint? *Answer*, $252.

9. A Vermonter, being in Boston, 80 miles from home, sold his dog and returned. At 6 o'clock the following night the dog left his new master, and at 6 o'clock the next morning stood at the door of his former master. How many steps did he take of 8 inches each?

10. What will the plastering of a room, 15 feet square, the walls 9 feet high, amount to, at 23 cts. a square yard, deducting for 2 doors, 7 feet by 3, and two windows, 5 feet by 3. *Answer*, $17,71.

11. How much time would a person lose in 20 years, by lying in bed ½ hour later every day than he ought? *Answer*, 152 days, 4 hours, 30 minutes.

12. How many cords of wood would a man draw in 6 weeks, drawing 4 loads a day, and 6½ cord feet at a load? *Answer*, 117 cords.

13. A merchant failing in trade, owes A £15, 7s. 9d.; B £69, 11s. 6d.; C £102, 16s., 11d.; D £41, 19s. 10d.;

E £139, 17s. 5d. His whole estate is valued at £300. How much does he owe more than he is worth?
Answer, £69, 13s. 5d.

14. How many shingles will cover the roof of a factory 100 feet in length, one side of the roof being 40 feet in width, if 4 shingles in width cover 2 feet in length, and 2 courses make a foot? *Answer*, 32,000.

15. How many boxes, each 12 lbs., can be filled from a hogshead of sugar containing 7½ cwt. *Answer*, 70.

16. In 46 bales of cloth, each containing 24 pieces, and each piece 42 ells Flemish, how many yards?
Answer, 34776 yards.

17. The sun travels through 6 signs of the zodiac in half a year? How many degrees, minutes and seconds?
Answer, 180 deg., 10800 m., 648000 sec.

18. How many English crowns, at 6s. 8d. each, in 10 English guineas, at 28s. each, and 24 pistoles, at 22s. each?
Answer, 121c. 1s. 4d.

19. The forward wheels of a waggon are 14½ feet in circumference, and the hind wheels 15¾ feet. How many more times will the forward wheels turn round than the hind wheels, in running from Concord to Boston, the distance being 60 miles? *Ans.*, 1734, rejecting fractions.

COMPOUND ADDITION.

1. A boy bought a slate for 4d., a book for 8d. What did both cost? *Answer*, 1s.

2. If I buy one book for 2s. 4d., another for 4s. 8d., what do I pay for both? *Answer*, 7s.

3. If a boy pay 4s. 8d. for a sled, and 5s. for a waggon, what does he pay for both? *Answer*, 9s. 8d.

4. How many shillings in 4d. 8d. 9d. 3d. 6d.? *Ans.* 2s.6d.

5. How many pounds are 8s. 7s. 4s. 3s. 9s. 5s. £1, 16s.

6. How many yards are 3 feet, 4 feet, 5 feet, 6 feet?
Answer, 6 yards.

7. Bought two pieces of cloth, one 10 yards, 1 foot, the other 12 yards, 2 feet. What was the length of both pieces? *Answer*, 23 yards.

8. What is the amount of £1, 4s. 2d. 3grs., and £10, 8s. 3d. and 2 grs.? *Answer*, £11, 12s. 6d. 1gr.

9. Add £4, 5s. 6d. 3grs., and £5, 17s. 7d. 2grs.

Operation.

£.	s.	d.	qs.	
4	5	6	3	
5	17	7	2	
10	3	2	1	*Ans.*

In adding the first column, or column of farthings, we find the amount to be 5 farthings. Now as 4 farthings are equal to 1 penny, we write the 1 farthing over, in the line of farthings, and carry the 1 penny to the column of pence. One to 7 is 8, and 6 are 14d. In 14d. there is 1 shilling, and 2 pence over, which we write in the column of pence, carrying the 1s. to the column of shillings. One added to 17 is 18 and 5 are 23. In 23 shillings there is £1, and 3s. over, which we write in the column of shillings, and carry 1 to the column of pounds.

From the foregoing questions and illustration we derive the following Definition and Rules.

COMPOUND ADDITION

Is the adding of numbers of different denominations. By different denominations is meant a different name—as shillings, pence, farthings, &c. Were the numbers given to be added, all pence, or all farthings, it would be but one denomination.

RULE.

1. Write numbers of the same denomination directly under each other, pounds under pounds, shillings under shillings, &c.

2. Begin to add at the right hand column, observing to carry one for as many in that column as make one in the next left hand column.

Proof—the same as in addition of simple numbers.

EXAMPLES.

1. Bought 4 books at the following prices, viz.: £1, 4s. 6d.; £2, 3s. 8d.; £2, 19s. 11d.; 2s. 3d. 2qr. To what did they amount? *Answer*, £6, 10s. 4d. 2qr.

2. Add the following numbers: £46, 26s. 7d. 3qr.; £49,18s.5d. 1qr.; £57, 17s. 9d. 2qr.; £102, 19s. 10d. 1qr. *Answer*, £258, 2s. 8d. 3qr.

3. Add $286, 12c. 6m.; $347, 20c. 4m.; $119, 18c. 7m; $542, 93c. 9m.; $314, 89c. 1m. *Ans.* $1610, 34c. 7m.

QUESTIONS. 1. What is Compound Addition? 2. Rule? 3. What is meant by different denominations?

6oz 16pwt.

13pwt 4gr.

gr.
13
12
11
9
15

gr.
19
5
13
16

gl.
2
3
1
1
2
3

Ale and Beer Measure.

hhd.	*gal.*	*qt.*	*pt.*	*hhd.*	*gal.*	*qt.*	*pt.*
102	21	2	1	171	29	1	1
201	39	3	0	169	49	3	0
310	42	2	0	289	38	1	1
412	38	1	1	169	42	1	1
121	39	2	1	128	31	2	1

17. Add 49 bushels, 3 pecks, 4 quarts, 1 pint; 39bu. 1pk. 5qt. 1pt.; 59bu. 2pk. 3qt. 0pt.; 40bu. 7pk. 2qt. 1pt.; 150bu. 0pk. 6qt. 1pt.; 69bu. 1pk. 2qt. 0pt.

18. Add 360 degrees, 15 miles, 6 furlongs, 16 poles, 13 feet, 6 inches; 240deg. 19m. 5fur. 29p. 11ft. 5in. 2b.; 159deg. 51m. 7fur. 32p. 14ft. 7in. 2b.; 201deg. 63m. 3fur. 15p. 12ft. 9in. 2b.

19. Add 971 miles, 6 furlongs, 11 poles, 3 yards, 1 foot; 239m. 5fur. 9p. 2yd. 2ft.; 269m. 7fur. 31p. 1yd. 2ft.; 67m. 6fur. 9p. 2yd. 2ft.; 691m. 5fur. 8p. 2yd. 2ft.

20. Add 69 acres, 2 roods, 1 rod; 76acr. 3ro. 39rd.; 88acr. 1ro. 32rd.; 150acr. 3ro. 29rd.

21. Add 150 years, 221 days, 13 hours, 31 minutes, 29 seconds; 230yr. 300d. 23h. 49m. 59s.; 191yr. 149d. 21h. 39m. 23s.; 359yr. 75d. 23h. 59m. 19s.

COMPOUND SUBTRACTION.

1. If a picture book cost 4d., and a spelling-book 11d., how much more does one cost than the other?

2. James bought a book for 9d., and sold it for 1s. How much did he gain by the bargain?

3. From 2s. 6d., take 1s. 8d.

4. From 8s. 9d. 3qrs., take 6s. 8d. 2qrs.

5. From 4qts., take 3pts.

6. If a bushel of rye be worth 7s. 6d., and a bushel of corn 6s. 4d., how much more is the rye worth than the corn?

7. How much more is wheat worth at 9s. 8d. per bushel, than corn at 7s. 6d. per bushel?

8. How much more is 2 bushels 2 pecks, than 1 bushel 3 pecks?

9. From £29, 9s. 6d. 3qrs., take £23 10s. 7d. 2qrs.

Operation.

£	s.	d.	qrs.
29	9	6	3
23	10	7	2
5	18	11	1

In this example, we write the difference between 2 and 3 farthings in the line of farthings, and proceed to the column of pence; we carry none, because we borrowed none—but 7d. from 6d. cannot be obtained; we therefore borrow as many pence as make a shilling, and say 7 from 12—the remainder 5, we add to 6 in the upper line, and write eleven in the column of pence. We now carry one to the column of shillings, which is equal to the 12 pence we borrowed, and say 11 from 9, which cannot be obtained; again we must borrow as many of the denomination we have to subtract, as make one of the next higher, which is 20s., and say 11 from 20 and 9 remains, which added to 9 in the upper line is 18, which must be written in the column of shillings. Lastly, the 20s. which we borrowed, we pay by carrying one to the line of pounds, which must be subtracted as in simple subtraction. Hence—

COMPOUND SUBTRACTION

teaches to find the difference between two compound sums, or quantities.

RULE.

1. Write the less number under the greater, so that numbers of the same denomination may stand directly under each other.

2. Begin to subtract with the lowest denomination, and take the lower line from the one above it; proceed in this way with all the denominations.

3. Should the numbers in the upper line be less than the one standing under it, borrow as many units as make 1 in the next higher denomination.

4. From the units borrowed, subtract the lower number, and to the difference add the upper number; write their sum under the figures subtracted, observing to carry 1 to the next left hand column.

Proof—the same as Simple Subtraction.

QUESTIONS. 1. What does Compound Subtraction teach? 2. Rule? 3. If the numbers in the upper be less than the one above, how may you proceed?

EXAMPLES.

Troy Weight.

lbs.	*oz.*	*pwt.*	*gr.*
91	10	19	21
87	11	15	19

lbs.	*oz.*	*pwt.*	*gr.*
39	11	14	20
37	11	15	19

Avoirdupois Weight.

Ton.	*cwt.*	*qr.*	*lbs.*	*oz.*	*dr.*
122	11	3	22	13	12
110	13	2	23	14	13

Ton.	*cwt.*	*qr.*	*lbs.*
39	11	14	20
37	9	15	19

Apothecaries' Weight.

lbs.	*oz.*	*dr.*	*scru.*	*gr.*
21	10	7	2	16
19	9	6	1	17

lb.	*oz.*	*dr.*	*scru.*	*gr.*
33	9	6	1	13
29	7	7	0	14

Wine Measure.

gal.	*qt.*	*pt.*	*gs.*
77	2	29	3
59	3	49	2

hhd.	*gal.*	*qt.*	*pt.*	*gs.*
600	3	59	2	1
459	3	47	3	0

Ale and Beer Measure.

hhd.	*gal.*	*qt.*	*pt.*
981	49	1	0
392	51	3	1

hhd.	*gal.*	*qt.*	*pt.*
1000	37	3	0
999	49	2	1

11. From 31 tuns, 3hhd. 15gal., take 29 tuns, 2hhd. 26gal.
12. From 39yds. 3qr. 2na., take 27yds. 2qr. 3na.
13. From 127E. E. 3qr. 2na., take 121E. E. 4qr. 3na.
14. From 247E. Fl. 0qr. 2na., take 159E. Fl. 2qr. 1na.
15. From 671E. Fl. 4qr. 3na., take 582E. Fl. 5qr. 2na.
16. From 971 mi. 6 fur. 11 p. 3 yds. 1ft., take 439 mi. 5 fur. 12 p. 4 yds. 2 ft.
17. From 69 acr. 2 ro. 31 rd., take 49 acr. 3 ro. 37 rd.
18. From 150 yrs. 221 d. 13 h. 31 m. 29 s., take 130 yrs. 129 d. 14 h. 39 m. 41 s.

19. From 260deg. 15 mi. 5 fur. 16 p. 13 ft., take 159 deg. 17 m. 6 fur. 17 p. 12 ft.

20. From 240° 49′ 31″, take 159° 59′ 41″.

21. From 9s. 21° 31′ 42″, take 7s. 22° 36′ 37″.

22. A note dated Feb. 3d, 1826, was paid March 12th, 1837; how long was it from the first date until it was paid?

The time from one date to another may be found by subtracting the former date from the latter, observing to number the months in their order; thus, Jan. 1st month, Feb. 2d month, &c.

Operation.

A. D., 1837	3d mo.	12th day.
A. D., 1826	2d mo.	3d day.
Answer, 11 ys.,	1 mo.,	9 days.

NOTE. The month, in casting interest, is reckoned 30 days.

23. What is the time from June 3d, 1835, to July 15th, 1837? *Ans.*, 2 yrs. 1 m. 12 d.

24. The latitude of a certain place is 42° 50′ north, that of another place is 39° 37′—what is the difference of latitude? *Answer*, 3° 13′.

25. What is the difference of longitude between 39° 40′, and 29° 49′ west? *Answer*, 9° 51′.

As every circle, whether greater or less, is divided into 360 equal parts, or degrees, consequently the circle, described by the revolution of the earth on its axis every 24 hours, contains 360 equal parts, or degrees—and as 360 degrees are described in 24 hours, it is plain, that, in 1 hour, $\frac{1}{24}$ of 360, or 15 degrees, would be described; and, also, if 15 degrees be described in 1 hour, or 60 minutes, it is equally plain, that 1 degree would be described in $\frac{1}{15}$ of 60 m., or in 4 m., and 1 minute of a degree in 4 seconds. Hence, to reduce longitude to time, we have the following

RULE.

Multiply the longitude in degrees and minutes by 4, and we have the time in minutes and seconds.

EXAMPLE.

Reduce 14° 15′ to time.

14° 15′
4
57′ 0″ *Ans.*

QUESTIONS. 4. How is the time from one date to another found? 5. How many degrees in a circle? 6. How many degrees does the earth describe in one hour, in its revolution round the sun?

To find the difference of time between any two places, having the time of one place given, and their difference of longitude. Reduce the longitude to time, and add it to the given time, if the longitude of the place whose time is required, be east of the place whose time is given; and subtract it if the longitude be west.

Note. The reason of this is, because the farther east we go, the later is it in the day, and the farther west, the earlier in the day. That is, when it is 12 o'clock at noon, in London, 15 degrees east of London it would be 1 o'clock, P. M. and 15 degrees west of London it would be but 11 o'clock, A. M.

27. When it is 12 o'clock in London, what is the hour in Boston, 70 degrees west longitude from London? *Ans.* 7 o'clock 20 m.

28. When it is 12 o'clock in Boston, what is the time in London, lon. 70° east? *Ans.*, 4 o'clock 40 m.

COMPOUND
ADDITION and SUBTRACTION.

1. A man bought a horse for £32 10s. and a pair of oxen for £24, 11s. 6½d., how much did both cost?

3. If I purchase a farm for £1092, 4s. 8d., for how much must I sell it to gain £57 19s. 8d.?

5. A pipe of wine sprang a leak and leaked out 31 gal. 1 qt. 1 pt. and there remained 86 gal. 2 qt. 1 pt., how many gallons were there at first?

7. There was a silver tankard which weighed 4 lbs. 3 oz.—the lid weighed 6 oz. 4 pwt. 6 gr., how much did both weigh?

2. If a pair of oxen and a horse cost £57, 1s. 6d. 2qr. and the horse cost £32, 10s., what was the cost of the oxen?

4. If I sell a farm for £1150, 4s. 4d. and gain £57 19s. 8d. by the bargain, what did the farm cost?

6. From a pipe of wine containing 118 gallons, there leaked out 31 gal. 1 qt. 1 pt., how many remained?

8. If the weight of a silver tankard and lid be 4 lbs. 9 oz. 4 pwt. 6 gr. and the lid alone weigh 6 oz. 4 pwt. 6 gr., what was the weight of the tankard?

Questions. 7. In one minute? 8. In one second? 9. Rule for finding the difference of time between two places, the longitude being known?

9. A merchant bought a quantity of sugar, sold 9 cwt. 3 qr. 25 lbs., had 7 cwt. 2 qr. 17 lbs. left, how much did he buy?

11. From a piece of cloth was sold 6 yds. 2 qr. and there remained 32 yds. 2 qr. 2 na., how much was there at first?

13. A farmer has 2 mowing fields, one contains 18 acres, 3 ro., the other, 12 acr. 2 ro. 24 rds., how many acres in both?

15. A note dated July 20, 1834, was paid in 9 mo. 46d., at what time was it paid?

10. A merchant bought 17 cwt. 2 qr. 14 lbs. of sugar, sells 9 cwt. 3 qr. 25 lbs., how much has he left?

12. If from a piece of cloth containing 39 yds. 2 na. were sold 6 yds. 2 qr., how many remained?

14. A farmer has 2 mowing fields containing 31 acr. 1 ro. 24 rds.; one contains 12 acr. 2 ro. 24 rds., how many acres did the other contain?

16. A note dated July 20, 1834, was paid June 6, 1835, how long was it on interest?

COMPOUND MULTIPLICATION and DIVISION.

1. If a bushel of oats cost 3s. 6d., how much will two bushels cost?

3. How much must be paid for 4 books, at 4s. 3d. each?

5. What will 5 yards of cloth cost at 3s. 8d. per yard?

7. How much beer in 8 bottles, each containing 2 qt. 1 pt. 2 gs.?

9. If one gallon of molasses cost 2s. 8d. 3qr., what will 8 gallons cost?

s. *d.* *qr.*
2 8 3
8
———
Ans. 1 1 10 0]

If one gallon cost 2s. 8d. 3qr. it is evident that 8 gallons will cost 8 times as much. We begin to multiply with the

2. If 2 bushels of oats cost 7 shillings, how much is it per bushel?

4. If 4 books cost 17s., what will 1 book cost?

6. If 5 yards of cloth cost 18s. 4d., how much is it per yard?

8. If 8 bottles contain 22 quarts, how much does one contain?

10. If 8 gallons of molasses cost £1 1s. 10d., what cost 1 gallon?

£ *s.* *d.* *qr.*
8) 1 1 10 0
———
Ans. 0 2 8 3]

If the price of 8 gallons be divided into 8 parts, it is evident, that one of these parts would be the lowest denomination, which is farthings. 8 times 3qr. are 24qr.=6d. 0qr. Place a cipher in the column of farthings, and proceed to multiply the column of pence, reserving the 6d. found in 24 qr. to be added—8 times 8d. are 64 d., and 6d. added are 70d.=5s. 10d. Write the 10d. in the column of pence, and reserve the 5s. to be added to the column of shillings. Lastly, 8 times 2s. are 16s. and 5s. added are 21s.= £1, 1s. and 10d., the answer.

COMPOUND MULTIPLICATION

is when the multiplicand consists of different denominations.

RULE.

Multiply the price by the quantity. When the quantity does not exceed 12, set down the price of one yard, one pound, or one gallon,&c. and the quantity under the lowest for a multiplier, observing to carry as in Compound Addition.

EXAMPLES.

1. What will 9 yards of cloth cost at 5s. 6d. per yd. ?

price of one gallon. Thus, 1 pound divided by 8, gives a cipher as a quotient figure, which must be written under the column of pounds, and 1 pound remains, which must be reduced to shillings—1× 20=20s. and 1s. added=21s. Dividing 21 shillings by 8, we have 2, as a quotient figure, and 5s. remainder, which reduced to pence 5×12=60 and 10d. added=70d., which divided by 8=8d and 6d over; reduce 6d.to qr.,6×4=24qr. divided by 8=3qr. we have then £0 2s. 8d. 3qr., the answer.

COMPOUND DIVISION

is when the dividend consists of different denominations.

RULE.

Divide the price by the quantity. When the quantity does not exceed 12, divide by the whole quantity at once. Divide the highest denomination by the divisor—then, multiply the remainder,if any, by as many of the next lower denomination as make one of that, adding to the product the next lower denomination; divide this sum by the given divisor, and so proceed.

EXAMPLES.

2. If 9 yards of cloth cost £2 9s. 6d., what will 1 yard cost ?

QUESTIONS. 1. What is Compound Multiplication ? 2. Compound Division ? 3. Rule for Compound Multiplication ? 4. Rule for Compound Division ? 5. How do you proceed when the multiplier is a composite number ? 6. When the Divisor is a composite number ?

3. What will 8 cwt. of cheese cost at £1, 10s. 5d. per cwt ?

5. What will 24 yards of cloth cost at 15s. 3d. per yd?

s. *d.*
15 3
6
4 11 6
4
18 6 0
[*Ans.*]

When the multiplier is greater than 12, and is a composite number multiply by its component parts, as in the last example, 6×4=24.

7. What is the weight of 56 casks of raisins, each weighing 1 cwt.2 qrs. 12 lbs.?

9. How much will 66 acres of land come to at £7, 9s. 6d. per acre?

11. What will 108 boxes of sugar weigh, each weighing 2 cwt. 1 qr. 14 lbs. ?

13. What will 112 yds of cloth cost at £1, 10s. 6d. per yard?

15. How much cloth will be required to make 121 coats, if, to make one, it requires 3 yds. 3 qrs. 3 na. ?

17. What is the value of 336 yards of cloth, at 2s. 5d. per yard?

19. What will 153 barrels of sugar weigh, each barrel weighing 3 cwt. 1 qr. 14 lbs.?

As 153 is not a composite number, we will first find the weight of 100, then of 50, then of 3; the several pro-

4. If 8 cwt. of cheese cost £12 3s. 4d., what is it per cwt. ?

6. If 24 yds. of cloth cost £18 6s., what will one yard cost?

£ *s.*8 *d.*
6)18 6 0
4) 3 1 0
0 15 3
[*Answer.*]

When the divisor is greater than 12, and is a composite number, divide by its component parts.

8. If 56 casks of raisins weigh 90 cwt., what is the weight of one cask?

10. If 66 acres of land cost £493, 7s., what will 1 acre cost?

12. If 108 boxes of sugar weigh 256 cwt. 2 qrs., what is the weight of one cask?

14. If 112 yards of cloth cost £170, 16s, what is it per yard?

16. If it take 476 yds. 1 qr. 3 na. to make 121 coats, how much will it require to make one?

18. If 336 yards of cloth cost £40, 12s., what is the cost of one yard ?

20. If 153 barrels of sugar weigh 516 cwt. 1 qr. 14 lbs., what is the weight of one barrel?

When the divisor is not a composite number, divide by the whole divisor at once,

---

QUESTIONS. 8. How do you proceed when the multiplier is greater than 12, and not a composite number? 9. How, when the divisor is not a composite number?

ducts added will be the answer. Thus—

```
cwt. qr. lbs.    cwt. qr. lbs.
  3  1  14×3=10  0  14
        10
───────────── cwt. qr. lbs.
 33  3   0×5=168  3  0
        10
─────────────
337  2   0  weight of 100
168  3   0  weight of 50
 10  0  14  weight of 3
─────────────
516  1  14  weight of 153.
```

The above may be given in the form of a rule. When the multiplier is not a composite number, and is hundreds, multiply by 10, and this product by 10, which will give the product of 100, and this by the number of hundreds. For tens, multiply the product of ten by the number of tens; for units, multiply the multiplicand. The several products added will be the answer sought.

after the manner of Long Division.

Thus, taking the last question, cwt. qr. lbs.

```
153)516  1  14(3 cwt.
    459
    ───
     57
      4
    ───
153)229(1qr.
    153
    ───
     76
     28
    ───
    622
    152
    ─────
153)2142(14lbs.
    153
    ───
    612
    612
```

The divisor 153 is contained in 516 three times, and there is a remaind'r of 57. That is, if 153 barrels weigh 516 cwt., 1 barrel weighs 3cwt. and 57 remainder, which are parts of a cwt., and must be reduc'd to quarters, the next lower denomination; therefore, multiply 57 by 4, and to the product add the one quarter, and divide the amount 229 by 153. We now have 1 qr. as the quotient, and a remainder of 76, which must be reduced to pounds by multiplying it by 28, and adding the 14 lbs. to the product.—Again, dividing by 153 we have 14 lbs. as the quotient. The several quotients, 3 cwt. 1 qr. 14 lbs., are the answer.

21. How much will a man spend in a year, if he spend 4d. a day?

23. What is the value of 1900 yards of linen, at 5s. 8½d. per yard?

22. If a man in one year spend £6 1s. 8d., how much will he spend in a day?

24. If 1900 yards of linen cost £542 5s. 10d., what will 1 yard cost?

25. What will 68 hogsheads of lime cost, at £1, 1s. 6¼d. per hhd.?

27. What is the value of 26 yards of silk, at 9s. 6½d. per yard?

29. How many gallons of beer in 14 bottles, each containing 3 qts. 1 pt. 1 gill?

31. What is the weight of 6 chests of tea, each weighing 3 cwt. 2 qrs. 9 lbs.?

33. How many acres in 9 fields, each containing 12 acr. 2 ro. 25 rds.?

35. How many cords of wood in 37 piles, each concontaining 8 cords, 28 ft.?

37. How much will 17 casks of nails weigh, each weighing 1 cwt. 2 qrs. 16 lbs. 3 oz.?

39. How many bushels of apples can be put into 125 barrels, each containing 3 bu. 1 pk. 5 qts.?

41. If a ship sail 2 deg. 30 m. 10 sec. in 1 day, how far will she sail in 30 days?

43. If 3 men build 14 rds, 8 feet of wall in one day, how much will they build in 26 days.

45. If 1 yd. of cloth cost £2, 2s. 6d, what will 229 cost?

47. The moon passes thro' 1 sign of the zodiac in 2d. 6h. 38m. 34 sec. In what time does it pass through 12 signs?

49. If one gallon of mo-

26. If 68 hhds. of lime cost £73 3s. 5d., what is it per hogshead?

28. If 26 yards of silk be worth £12, 8s. 1d., what will 1 yard be worth?

30. If 14 bottles of beer contain 12 gal. 2 qts. 1 pt. 2 gills, how much does 1 bottle contain?

32. If 6 chests of tea weigh 21 cwt. 1 qr. 26 lbs., what is the weight of 1 chest?

34. If in 9 fields there are 113 acr. 3 ro. 25 rds., how many in one field?

36. If 37 piles of wood contain 304 cords and 12 ft., how much in one pile?

38. If 17 casks of nails weigh 27 cwt. 3 qrs. 23 lbs. 3 oz., what will one cask weigh?

40. If 125 barrels contain 425 bush. 3 pks. 1 qt., how much does one contain?

42. If a ship sail 75 deg. 5 m. in 30 days, how far will she sail in one day?

44. If 3 men build 376 rods, 10 feet, in 26 days, how much do they build in 1 day?

46. Bought 229 yds. of cloth for £486, 12s. 6d.—What did it cost per yard?

48. If the moon pass thro' 12 signs of the zodiac in 27d. 7h. 42m. 48 sec., in what time does it pass through 1 sign?

50. If 1000 gallons of mo-

lasses cost 4s. 2d. 2 qrs., what will 1000 gallons cost?

51. If one pound of tea cost 8s. 5d. 2qrs. what will 108 lbs. cost?

53. If one quintal of fish cost 23s. 9d., what will 345 quintals cost?

lasses cost £210 8s. 4d. what is it per gallon?

52. If 108 pounds of tea cost £45 13s. 6d., what will one pound cost?

54. If 345 quintals of fish cost £409 13s. 9d., what was it per quintal?

◆

SUPPLEMENT TO COMPOUND NUMBERS.

1. What is the weight of two pieces of gold, one weighing 1 lb. 0 oz. 6 pwt. 4 grs.; the other, 2 lbs. 3 oz. 8 pwt. 16 grs? *Answer*, 3 lbs. 3 oz. 14 pwt. 20 grs.

2. A man has one wedge of gold, weighing 25 lbs. 3 oz. 12 pwt., and another weighing 1 lb. 11 oz. 12 pwt. 7 grs. What is the weight of the two?

Answer, 27 lbs. 3 oz. 4 pwt. 7 grs.

3. A silver-smith had a quantity of silver, weighing 21 lbs. 9 oz. After refining it by melting, it weighed 15 lbs. 10 oz. 11 pwt. 19 grs. What was lost by melting?

Answer, 5 lbs. 10 oz. 8 pwt. 5 grs.

4. What is the sum and difference of 3 lbs. 10 oz., and 2 lbs. 11 oz. 7 pwt. 4 grs.

Answer, { Sum: 6 lbs. 9 oz. 7 pwt. 4 grs.
Difference: 10 oz. 12 pwt. 20 grs.

5. What will 13 lbs. of coffee cost at 1s. 2d. 3 qrs. per pound? *Answer*, 15s. 11d. 3 qrs.

6. What will 47 yds. of cloth cost at 17s. 9d. per yard?

Answer, £41 14s. 3d.

7. How much will 10 cwt. of lead cost, at 7d. per lb.?

Answer, £32 13s. 4d.

8. What is the value of 7 cwt. of sugar, at 4¾d. per lb.?

Answer, £15 10s. 4d.

9. What is the weight of 4 hogsheads of sugar, each weighing 7 cwt. 3 qrs. 19 lbs.?

Answer, 31 cwt. 2 qrs. 20 lbs.

10. Bought 1½ doz. large silver spoons, each weighing 3 oz. 5 pwt.; two doz. tea-spoons, each weighing 15 pwt. 14 grs.; three silver cups, each weighing 9 oz. 7 pwt.; two silver tankards, each 21 oz. 15 pwt.; 6 silver porrin-

gers, each 11 oz. 18 pwt. What is the weight of the whole? *Answer*, 18 lbs. 4 oz. 3 pwt.

11. If 6 ells cost £5 17s. 6d. what will one ell cost? *Answer*, 19s. 7d.

12. What must a man spend per month, to spend £17 14s. 6d. in a year? *Answer*, £1 9s. 6½d.

13. If 8 cwt. of cocoa cost £15 17s. 4d., what is it per pound. *Answer*, 4d. 1 qr.

14. If 132 bush. of oats cost £20 12s. 6d. what is the cost of 1 bushel? *Answer*, 3s. 1d. 2 qr.

15. If 147 bushels of corn cost £47 12s. 5d., what does it cost per bushel? *Answer*, 6s. 5d. 3 qr.

16. If one acre produce 152½ bushels of oats, how much will a square rod produce? *Ans.* 3 pks. 6 qts. 1 pt.

17. How much wood in 11 piles, each containing 120 cords, 7 cord-feet, 11 solid feet?

Answer, 1330 cords, 4 cord-feet, 9 solid ft.

18. Multiply £86 12s. 6d. by 9; divide the product by 6; multiply the quotient by 4; divide the product by 12, and give the result.

19. If it take a printer 297h. 59m. 24sec. to set 108 pages, how long will it take him to set one page?

Answer, 2h. 45m. 33sec.

20. A person wishes to draw a pipe of wine into bottles, containing a quart, 2 quarts, 1½ pint, ½ pint, of each an equal number; how many must he have?

When it is required to find how many times several quantities, each an equal number, may be had in a given quantity—

RULE. Reduce the given quantity to the lowest denomination mentioned for a dividend, and each of the other quantities to the same denomination, and add them together for a divisor. The quotient will be the answer.

1 quart = 8 gills.
2 quarts = 16 gills.
1½ pint = 6 gills.
½ pint = 2 gills.

32 gills.

The 1 pipe reduced to gills equals 4032 gills, and 4032÷32=126 bottles, the *Answer*.

21. How many bushels, half bushels, and peck baskets, of each an equal number, will it take to contain 100 bushels? *Answer*, 57, and 8 qts. over.

QUESTIONS. 10. What is the Rule for the 20th example?

22. There are four fields, one containing 10 acres, 2 roods; another 9 acres; another 11 acres, 3 roods; another, 6 acres, 3 roods, 30 rods. How many shares, of 65 rods each? *Answer*, 94.

23. A man left $1043,28 to be divided as follows: His wife is to have two thirds; of the other $\frac{1}{3}$, his sister is to have $\frac{1}{2}$, and the remainder is to be divided between two nephews and nine distant relatives. To one nephew, he gives 3 shares—to the other, 2—to each of the relatives 1 share each. What is the share of each respectively?

Answer,

Wife:	$695,52
	173,88
Nephew:	37,26
do.	24,84
Relatives:	12,42

24. What will 156 acres of land cost at £5 6s. 9d. 2qr. per acre? *Answer*, £832 19s. 6d.

25. A values a piece of land at $120, B at $100, C at $110. What is the average judgment?

A 1 $120
B 1 100
C 1 110
3) 330
$110 *Answer.*

The average is found by dividing the sum of the several judgments by the number of judges.

26. Two gentlemen wished to exchange vehicles. One was a gig, the other was a waggon; but not being able to agree as to the conditions, referred the matter to A, B and C, who decided as follows. A said the owner of the gig should pay the owner of the waggon $20, and B. said he should pay $15; but C said the owner of the waggon should pay the owner of the gig $10. What is the average judgment? *Answer*,—the owner of the gig must pay 8\frac{1}{3}$.

In cases where the judgment of the referees is part on one side of the question, and part on the other, subtract one side from the other, and divide the remainder by the number of referees, and the quotient will be the answer.

27. A and B wish to exchange watches, but cannot agree upon the difference. They refer the matter to C, D and E, and agree to abide by their decision. C gives his

opinion that A should give B $3. D thinks the difference in B's favor is $4 ; but E takes the other side of justice, and says B should pay A at least $1. What is the average judgment? *Answer*, A must pay $2.

FRACTIONS.

Fractions are parts of a unit, or of a whole number. If a unit, or any particular thing, be divided into parts, these parts are called *fractions.* If a thing be divided into two equal parts, these parts are called halves; if into three equal parts, they are called thirds; if into four, fourths, or quarters, &c.

The dividing figure generally gives name to the parts into which a thing is divided. If the divisor be 2, the quotient will be one half of the dividend; if the divisor be 3, the quotient will be one third; if the divisor be 4, the quotient will be one fourth, &c.

Fractions are expressed by two figures, written one above the other, with a line drawn between them. The figure below the line (which is the same as the dividend in Division) shows into how many parts the integer, or whole number, is divided, and is therefore called the denominator. The figure above the line denotes the number of parts contained in the fraction, and is thence called the numerator. Thus $\frac{1}{2}$ (one half)—the figure above the line shows that one of those parts is contained in the fraction ; $\frac{3}{4}$, the 4 shows that the whole number is divided into four parts ; and the 3, that three of these parts are contained in the fraction. If $100 be divided into 5 parts, one of those parts will be $\frac{1}{5}=20$ dolls ; two of those parts, $\frac{2}{5}=40$ dolls. ; three parts, $\frac{3}{5}=60$ dolls. ; four parts, $\frac{4}{5}=80$ dolls. ; and five parts, $\frac{5}{5}=100$ dollars.

Fractions are of two kinds—Vulgar, or common ; and Decimal. They differ in the form of expression and mode of operation.

Questions. 1. What are Fractions? 2. If a thing be divided into two equal parts, what are these parts called? 3. Into four equal parts? 4. To what does the dividing figure give name? 5. How are fractions expressed? 6. What is the figure below the line called? 7. What is the figure above the line called? 8. and what does it show? 9. What does the denominator show in the fraction? 10. The numerator? 11. Fractions are of how many kinds, and what are they? 12. In what do they differ?

In Decimal Fractions, the unit, or integer, is divided into 10, 100, 1000, &c., equal parts; or the denominator is always 1, with as many ciphers annexed as the numerator has places.

In Vulgar Fractions the integer may be divided into any number of parts, and the denominator being always expressed, may be any thing but 1, with or without a cipher or ciphers annexed.

Vulgar Fractions are either proper, improper, compound or mixed.

1. *A proper Fraction* is one whose numerator is less than the denominator; as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c.

2. *An improper Fraction* is one whose numerator is equal to, or greater than the denominator; as $\frac{8}{8}$, $\frac{9}{8}$, $\frac{8}{7}$, &c.

3. *A compound Fraction* is a fraction of a fraction; as $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{4}{5}$, &c.

4. *A mixed number* is a whole number and fraction written together, as $2\frac{1}{3}$, $14\frac{6}{7}$, &c.

5. *A common divisor, or common measure* of two or more numbers, is a number which will divide each of them without a remainder—4 is the common measure of $\frac{12}{48}$.

6. *The greatest common divisor* of two or more numbers, is the greatest number which will divide those numbers without a remainder. Thus, 12 is the greatest common measure of $\frac{12}{48}$.

7. Two or more fractions are said to have a *common denominator*, when the denominator of each is the same.

8. *A common multiple* of two or more numbers, is a number which may be divided by each of those numbers without a remainder.

9. The *least common multiple* of two or more numbers, is the least number, which may be divided by those numbers without a remainder. Thus, 8 is the least common multiple of 8, 4 and 2.

10. A perfect number is equal to the sum of all its aliquot parts. Thus, 6 is equal to $3\times2\times1=6$.

QUESTIONS. 13. How is the unit divided in Decimal Fractions? 14. What is always the denominator? 15. In Vulgar Fractions, how is the integer divided? 16. What may the denominator be? 17. How are Vulgar Fractions divided? 18. What is a proper fraction? 19. An improper fraction? 20. A compound fraction? 21. A mixed number? 22. What is a common divisor, or common measure of two numbers? 23. The greatest common divisor? 24. When are fractions said to have a common denominator? 25. What is meant by a common multiple of two or more numbers? 26. The least common multiple? 27. What is a perfect number? 28. What is meant by the terms of a fraction?

11. The numerator and denominator of a fraction, taken together, are called the *terms* of the fraction.

12. A fraction is said to be in its lowest terms, when no number greater than 1, or unity, will divide the *terms* of the fraction without a remainder.

EXAMPLES.

1. If I divide an apple into 8 parts, by what fraction will one of those parts be expressed? 2 of those parts? 3—4—5—6—7—8? *Ans.* $\frac{1}{8}$, $\frac{2}{8}$, $\frac{3}{8}$, $\frac{4}{8}$, $\frac{5}{8}$, $\frac{6}{8}$, $\frac{7}{8}$, $\frac{8}{8}$.

2. If one be divided by 4, what will be the quotient? if by 6, what? if by 7? if by 8? if by 9? if by 10? if by 11? if by 12? *Ans.* $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{11}$, $\frac{1}{12}$.

3. If 2 be divided by 4, what will be the quotient? if by 6? by 7? by 8? by 13? *Ans.* $\frac{2}{4}$, $\frac{2}{6}$, $\frac{2}{7}$, $\frac{2}{8}$, $\frac{2}{13}$.

4. If a bushel be divided equally among 4 persons, what part of a bushel does each receive?

5. If 2 bushels of apples be divided equally among 4 persons, what will each receive?

6. If a bushel of corn be divided into 4 parts, what are those parts called? if into 5 parts? into 6? into 7?—8? 9?—10?

7. If I give away 6 quarts of nuts, what part of a peck is it? if 7? if 8? if 9?

8. How many of the four last questions are proper fractions? Are any improper, and which are they?

9. If I divide an apple into halves, and give away $\frac{1}{2}$ of $\frac{1}{2}$, what part do I give away? and what is the fraction by which it is expressed in the question?

10. How is the quotient of 4, divided by 3, expressed?

Questions similar to the foregoing, should be multiplied and proposed to the scholar, until his ideas of fractions are clear and correct.

To reduce Improper Fractions to Mixed Numbers, and Mixed Numbers to Improper Fractions.

IMPROPER FRACTIONS.

1. Change $\frac{67}{4}$ to a whole or mixed number.

$$\begin{array}{r} 4)67 \\ \hline 16\frac{3}{4} \end{array}$$

As the denominator of a fraction denotes the number of parts into which the unit

MIXED NUMBERS.

2. Change $16\frac{3}{4}$ to an improper fraction.

$$\begin{array}{r} 16\frac{3}{4} \\ 4 \\ \hline 67 \\ \hline 4 \end{array}$$

The scholar will perceive, that the mixed number, $16\frac{3}{4}$, was the quotient in the last question, of 64 divided by 4; and

---

QUESTION. 29. When is a fraction said to be in its lowest terms?

is divided, it is evident, that $\frac{67}{4}$ contains as many units, or wholes, as four is contained times in 67, which we find by trial, to be 16 times and $\frac{3}{4}$ of a time.

Hence the Rule—

To reduce an improper fraction to a whole, or mixed number.

Divide the numerator by the denominator, and the quotient will be the whole number; the remainder, if any, written over the denominator, must be placed at the right hand of the quotient.

EXAMPLES.

3. Change $\frac{42}{8}$ to a whole or mixed number.

5. In $\frac{59}{9}$, how many wholes?

7. In $\frac{159}{15}$ of a week, how many weeks?

9. Change $\frac{2233}{14}$ to a whole or mixed number?

11. In $\frac{3247}{16}$ of a day, how many days?

13. In $\frac{49364}{39}$ of a year, how many years?

15. In $\frac{17}{5}$ of a cent, how many half dimes?

17. Change $\frac{229}{4}$ to a whole or mixed number.

19. In $\frac{504}{7}$ of a minute, how many minutes?

let it be remembered that a mixed number is the quotient of a division, whose divisor is the denominator of the fraction; therefore in reducing a mixed number to an improper fraction, we only find the dividend.—(See p. 40.)

Hence the Rule—

To reduce a mixed number to an improper fraction.

Multiply the whole number by the denominator of the fraction, and to the product add the numerator; under the result, place the denominator of the fraction.

EXAMPLES.

4. Change $5\frac{2}{8}$ to an improper fraction.

6. In $6\frac{5}{9}$, how many ninths?

8. In $10\frac{9}{15}$ weeks, how many 15ths?

10. Change $159\frac{7}{14}$ to an improper fraction.

12. In $202\frac{15}{16}$ days, how many 16ths?

14. In $1265\frac{29}{39}$ years, how many 39ths?

16. Change $3\frac{2}{5}$ half dimes to the fraction of a cent?

18. Change $57\frac{1}{4}$ to an improper fraction?

20. In 72 minutes, how many 7ths?

To reduce a Fraction to its lowest terms.

1. Reduce $\frac{4}{8}$ to its lowest terms.

If $\frac{4}{8}$ of a bushel of apples were divided equally between two persons, it is evident that one person would receive $\frac{2}{8}$

QUESTIONS. 1. What is the rule for reducing an improper fraction to a whole or mixed number? 2. For reducing a mixed number to an improper fraction?

of a bushel. By dividing the numerator by 2, the fraction is diminished. If we divide the denominator by 2, it becomes $\frac{4}{4}$, and is therefore increased; but, if we divide both the numerator and denominator of $\frac{4}{8}$ by 2 it becomes $\frac{2}{4}$; if we divide the terms of the fraction again by 2 it becomes $\frac{1}{2}$, which is equal to $\frac{2}{4}$ or $\frac{4}{8}$, for in either case the numerator is one half of the denominator. Hence it appears that the value of a fraction is not affected by dividing or multiplying both the numerator and denominator by the same number. To reduce a fraction to its lowest terms, we have this Rule.

Divide both the numerator and denominator by any number that will divide both without a remainder; and so continue to do, until no number greater than 1 will divide them.

2. Reduce $\frac{84}{210}$ to its lowest terms.

$$7\left.\right\}\frac{84}{210}\left\{\right.{}^{6)}\frac{12}{30}=\frac{2}{5}\quad\textit{Answer.}$$

3. Reduce $\frac{160}{760}$ to its lowest terms. *Ans.* $\frac{4}{19}$.

4. Reduce $\frac{371}{1232}$, $\frac{77}{171}$, $\frac{100}{180}$, $\frac{83}{249}$, to their lowest terms.

Were the greatest number known which would divide the terms of the fraction, a simple division would at once reduce the fraction; but, as this is not the case, the greatest divisor may be found by the following

RULE.

Divide the denominator by the numerator, and if there be no remainder, the numerator will be that divisor; but if there be a remainder, divide the last divisor by the last remainder, and thus proceed until there be no remainder; and the last divisor will be the *greatest common measure sought.*

5. Reduce $\frac{16}{76}$ to its lowest terms.

Operation.

```
16)76(4
   64
   ——
   12)16(1
      12
      ——
       4)12(3
         12
         ——
```

We find by trial that 16 is contained in 76, 4 times and 12 remainder; hence 16 is not the common divisor; dividing the last divisor 16, by the remainder 12, we have the quotient 1, and 4 remainder: therefore 12 is not the common divisor. Had 12 divided 16 without a remainder, it is evident, that it would have been a divisor common to both terms of the fraction, because $76=16\times4+12$. It is plain, if 12 would divide 16 without a remainder, it would also divide 4 times $16+12$ without a remainder. Again, we find that 12, the last divisor, will contain 4, the last remainder, 3 times and no remainder. 4, is therefore the *greatest common divisor* of the terms of the fraction $\frac{16}{76}$, and $\frac{4}{19}$ is the answer in its lowest terms, for no number will divide 4 and 19 without a remainder.

6. Reduce $\frac{182}{196}$ to its lowest terms. *Ans.*, $\frac{13}{14}$.
7. Reduce $\frac{468}{1184}$ to its lowest terms. *Ans.*, $\frac{117}{296}$.
8. Reduce $\frac{6912}{32256}$ to its lowest terms. *Ans.*, $\frac{8}{14}$.
9. Reduce $\frac{36}{1296}$ to its lowest terms. *Ans.*, $\frac{1}{36}$.

If it be required to find the greatest common measure of more than two numbers, find the greatest common measure of two of them as before; then, of that common measure, and of one of the other numbers, and so on thro' the whole. The common measure last found, will be the one sought.

10. What is the greatest common measure of 48 and 192? *Answer*, 48.

11. Reduce $\frac{48}{192}$ to its lowest terms. *Answer*, $\frac{1}{4}$.

12. What is the greatest common measure of 35—42—63? *Answer*, 7.

◆

GENERAL RULE FOR THE MULTIPLICATION AND DIVISION OF FRACTIONS BY WHOLE NUMBERS —WHOLE NUMBERS BY FRACTIONS—FRACTIONS BY FRACTIONS.

Draw a perpendicular line, and place all those figures which are to be multiplied together for a numerator, or dividend, on the right of the line, and those figures which are to be multiplied together for a denominator, or divisor, on the left hand of the line. (See p. 43.) It will appear obvious to the scholar by the analysis under each of the following sections, that the numerators of fractions to be multiplied or divided, must be placed upon the right of the line, and their denominators on the left; and also that the numerators of fractions, by which the division is to be made, on the left. The question thus stated, equals on

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QUESTIONS. 3. What is the rule for reducing a fraction to its lowest terms? 4. Were the greatest number known which would divide the terms of the fraction, how might you proceed? 5. When this is not the case, how may the greatest divisor be found? 6. How is the common measure of more than two numbers found? 7. What is the rule for the multiplication and division of fractions, &c. by cancelling?

each side of the line may be crossed, as cancelling each other. (See p. 43.) When no two numbers remain, one on each side of the line, capable of being divided by any one figure, (See p. 90.) multiply the figures on the right of the line for a numerator, or dividend, and those on the left for a denominator, or divisor, and the result will be the answer in the lowest terms of the fraction.

Multiplication of Fractions by Whole Numbers.

1. If a man receive $\frac{1}{4}$ of a dollar for one day's work, what will he receive for two days' work?

It is evident, if a man receive $\frac{1}{4}$ of a dollar for 1 day's work, that he would receive for two day's work twice as much or $\frac{2}{4}=\frac{1}{2}$. If I multiply the numerator by 2, I give him $\frac{2}{4}$, or 2 quarters of a dollar; if I divide the denominator by 2, I give him $\frac{1}{2}=\frac{2}{4}$ or quarters. Hence, to multiply the numerator of a fraction is the same in effect, as to divide the denominator. If the numerator of $\frac{1}{2}$ be multiplied by 2, it becomes $\frac{2}{2}=1$; if the denominator be divided by 2, it becomes $\frac{1}{1}=1$. Therefore—

To multiply a Fraction by a Whole Number—Multiply the numerator, or divide the denominator; and the result will be the answer required.

2. If a pound of lead cost $\frac{1}{16}$ of a dollar, how much will 8 pounds cost?

Operation.

$$\begin{array}{r|l} & \cancel{8} \\ 2\ \cancel{16} & 1 \\ \hline 2 & 1=\frac{1}{2} \\ & \end{array}$$

As $\frac{1}{16}$, the cost of 1 pound, is to be multiplied by 8, the number of pounds; therefore, place the numerator of $\frac{1}{16}$, the multiplicand, and 8, the multiplier, on the right of the line; since 8, on the right is contained in 16, on the left, twice, cross the 8 and 16, and write 2 on the left of 16; we now have 1 on the right to be divided by 2 on the left, which is $\frac{1}{2}$, the answer.

3. What will 8 bushels of apples cost at $\frac{1}{2}$ a dollar per bushel? *Answer*, $4,00.

QUESTIONS. 8. When the question is stated, what is the method of procedure? 9. When no two numbers are left, one on each side of the line, capable of being divided by any one figure, what is to be done? 10. How do you multiply a fraction by a whole number? 11. Why, in example 2, are 8 and the numerator of the fraction placed on the right of the line?

4 If one man can plant $\frac{3}{4}$ of an acre in one day, how much could 12 men plant in the same time? *Answer*, 9 acres.

5. If one barrel of fish cost $6\frac{1}{4}$ dollars, what will 9 barrels cost? *Answer*, \$$56\frac{1}{4}$.

Note. Mixed numbers must be reduced to improper fractions.

6. If one chest of tea cost \$$25\frac{1}{4}$, what will 15 cost? *Answer*, \$$378\frac{3}{4}$.

7. If a man can walk $29\frac{1}{2}$ miles in one day, how far could he walk in 30 days? *Answer*, 885 miles.

8. What will 600 pounds of cotton cost, if 1 pound cost $9\frac{1}{2}$ cents? *Answer*, \$57.

Division of Fractions by Whole Numbers.

1. If a man receive $\frac{2}{4}$ of a dollar for two day's work, what does he receive per day?

We have seen that a fraction is multiplied either by multiplying its numerator, or dividing its denominator; then, as Division is the reverse of Multiplication, the reverse of the rule for Multiplication will be the rule for Division. If I divide the dollar into 4 parts, or quarters, and pay a man $\frac{1}{4}$, or quartor, it is the same as though I should divide it into 8 parts, or half quarters, and pay him $\frac{2}{8}$, or 2 half quarters$=\frac{1}{4}$—therefore,

To divide a Fraction by a Whole Number—Divide the numerator of the fraction by the whole number, when it can be done without a remainder, otherwise, multiply the denominator.

2. If 8 pounds of lead cost $\frac{1}{2}$ of a dollar, what is it per pound?

Operation.

$$\begin{array}{r|l} 2 & 1 \\ 8 & \\ \hline 16 & 1=\frac{1}{16}. \end{array}$$

As $\frac{1}{2}$ of a dollar, the whole cost, is to be divided by 8, the number of pounds, therefore write 1, the numerator of $\frac{1}{2}$, which is the dividend, on the right hand, and 8, which is the divisor on the left. The figure on the right being only a unit, cannot be reduced, therefore, the 2 and 8 on the left are to be multiplied together for a divisor, or denominator; the answer, then, is 1 divided by $16=\frac{1}{16}$.

3. If a pair of oxen plough in 4 days $\frac{28}{31}$ of a field, what part do they plough in one day? *Answer*, $\frac{7}{31}$.

Questions. 12. How do you divide a fraction by a whole number? 13. Why, in example 2d, are 8 and 2, the denominator of the fraction, placed on the left? 14. Multiplying the denominator of a fraction, is the same as what?

4. If 6 men earn $\frac{18}{28}$ of a guinea in a day, what part of a guinea does one man earn in the same time? *Ans.* $\frac{3}{28}$.

5. If 16 hats cost $\$64\frac{1}{2}$, what does one hat cost? *Answer*, $\$4\frac{1}{32}$.

6. What will 1 pipe of molasses cost, if 14 pipes cost $\$707\frac{1}{2}$? *Answer*, $\$51\frac{15}{28}$.

7. What will one yard of broadcloth cost, if 25 yards cost $\$150\frac{3}{4}$? *Answer*, $6,03.

8. What will one pound of rice cost, if 50 pounds cost $\$150\frac{3}{4}$? *Answer*, $\$3\frac{3}{200}$.

Multiplication and Division of Fractions by Whole Numbers.

MULTIPLICATION.	DIVISION.
1. If one dollar will buy $\frac{1}{18}$ of an acre of land, how much will 9 dollars buy?	2. If 9 dollars will buy $\frac{1}{2}$ of an acre, how much will 1 dollar buy?
3. If a man travel $\frac{2}{27}$ of a mile in one minute, how far will he travel in 12 minutes?	5. If a man travel $\frac{8}{9}$ of a mile in 12 minutes, how far will he travel in one minute?
5. If a man consume $\frac{5}{42}$ of a barrel of flour in one month, what will 7 men consume in the same time?	6. If 7 men consume $\frac{5}{6}$ of a barrel of flour in 1 month, how much will one man consume?
7. If $\frac{3}{14}$ of a box of glass cost one dollar, how many boxes will 21 dollars buy?	8. If 21 dollars will buy $4\frac{1}{2}$ boxes of glass, how much will 1 dollar buy?
9. If a pound of chocolate cost $\frac{5}{28}$ of a dollar, how much will 7 pounds cost?	10. If 7 pounds of chocolate cost $\frac{5}{4}$ of a dollar, what will 1 pound cost?
11. If a man can do $\frac{1}{12}$ of a piece of work in one day, how much could he do in 8 days?	12. If a man can do $\frac{2}{3}$ of a piece of work in 8 days, how much can he do in 1 day?
13. What will 16 yards of cloth cost, at $\frac{3}{4}$ of a dollar per yard?	14. If 16 yards of cloth cost $\frac{12}{1}$ of a dollar, what will one yard cost?
15. What will 40 yards of carpeting cost, at $\frac{7}{8}$ of a dollar per yard?	16. If 40 yards of carpeting cost $\frac{35}{1}$ of a dollar, what will 1 yard cost?
17. If 1 pint of wine cost $\frac{7}{96}$ of a dollar, how much will 12 quarts cost?	18. If 12 quarts of wine cost $1\frac{3}{4}$ dollar, what is it per pint?
19. Multiply $\frac{7}{8}$ by 11.	20. Divide $9\frac{5}{8}$ by 11.

Multiplication of Whole Numbers by Fractions.

1. If a barrel of flour cost $9, how much will $\frac{2}{3}$ of a barrel cost?

Had the cost of 2 barrels been required, the price of 1 barrel being given, we should multiply the price of 1 barrel by the number of barrels. The same is now to be done; that is, the price of one barrel is to be multiplied by the part or parts of a barrel taken. The product of any number multiplied by 1, is the same as the multiplicand; the product of any number greater than 1, is as many times greater than the multiplicand, as the multiplier contains units. The product of any number less than 1, is as much less than the multiplicand as the multiplier is less than a unit, or 1. Therefore, if we multiply 9 by $\frac{2}{3}$ of 1, the product will be $\frac{2}{3}$ of 9, or 6. Hence it appears, that the product of a whole number multiplied by a fraction, is the value of the fraction taken from it. If $9 be the cost of 1 barrel, then the quotient of $9 divided by 3 will be $3, the cost of $\frac{1}{3}$ of a barrel, and $3\times2=6$ the cost of $\frac{2}{3}$, therefore,

To multiply a Whole Number by a Fraction—Divide the whole number by the denominator of the fraction, and multiply the quotient by the numerator.

2. What will 16 yards of cloth cost at $\frac{3}{4}$ of a dollar per yard?

Operation.

$$\begin{array}{r|l} & \cancel{16}\ \ 4\times3=\$12 \\ \cancel{4} & 3 \\ \hline \end{array}$$

As 16, the number of yards, is to be multiplied by 3, the numerator of the fraction—the 16 and 3 fall on the right hand of the line, and 4, the divisor, on the left. 4 is contained in 16, four times; cross 4 and 16, and write 4, the quotient, on the right of 16—the 3 and 4 then multiplied give $12, the answer.

3. What will 40 yards of carpeting cost, at $\frac{7}{8}$ of a dollar per yard? *Answer*, $35.

4. What will 64 bushels of oats cost at $\frac{3}{8}$ of a dollar per bushel? *Answer*, $24.

5. What will 24 bushels of corn cost, at $\frac{5}{8}$ of a dollar per bushel? *Answer*, $15.

6. Multiply 21 by $\frac{3}{7}$, by $\frac{6}{7}$, by $\frac{1}{3}$. *Answer*, 9, 18, 7.

7. What is the product of 324 multiplied by $\frac{11}{12}$?

QUESTIONS. 15. What is the product of any number multiplied by 1? 16. How many times greater than the multiplicand is the product of a multiplier greater than 1? 17. How much less than the multiplicand is the product, when the multiplier is less than 1? 18. What is the product of a whole number multiplied by a fraction? 19. Rule?

8. What will 9 pounds of tea cost, at $\frac{27}{57}$ of a dollar per pound? *Answer*, \$4$\frac{1}{2}$.

9. What will 56 lbs. of butter cost, at $\frac{2}{8}$ of a dollar per pound? *Answer*, \$14.

10. What will 124 lbs. of sugar cost, at $\frac{2}{16}$ of a dollar per pound? *Answer*, \$15$\frac{1}{2}$.

11. Multiply 32 by $\frac{1}{4}$, by $\frac{2}{8}$, by $\frac{3}{8}$, by $\frac{7}{8}$.
Answer, 8, 8, 12, 28.

12. Multiply 224 by $\frac{1}{56}$. *Answer*, 4.

Division of Whole Numbers by Fractions.

1. If $\frac{2}{3}$ of a barrel of flour cost \$6, what will be the cost of 1 barrel?

As this question is the reverse of question 1st, in the preceding section, the reverse of the rule there given will be the rule for solving this question. Had \$6 been the cost of $\frac{1}{3}$ of a barrel, it is evident, that 3 times \$6, or 18, would have been the cost of $\frac{3}{3}$, or 1 barrel; but, as \$6 is the cost of $\frac{2}{3}$ of a barrel, if I multiply \$6 by 3, I shall have three times $\frac{2}{3}$, or $\frac{6}{3}=2$ barrels; if then $\frac{6}{3}$, or 2 barrels be divided by 2, I shall have the price of $\frac{3}{3}=\$9$, or 1 barrel. Hence, it appears that the quotient of a whole number divided by a fraction, will be as much greater than the dividend as the fraction is less than a unit, or 1.

RULE.

Multiply the dividend by the denominator of the fraction, and divide the product by the numerator.

Operation.

$$\begin{array}{r|l} & \cancel{6} \quad 3\times3=\$9 \ \textit{Ans.} \\ \cancel{2} & 3 \\ \hline & \end{array}$$

As 6, the dividend, is to be multiplied by 3, the denominator of the fraction, place 6 and 3 on the right of the line, and 2, the divisor, on the left. Then we say, 2 in 6, 3 times; cross 6 and 2, and multiply 3 by 3—3×3—\$9. *Ans.*

2. If $\frac{5}{6}$ of a ton cost \$15, how much will 1 ton cost?
Answer, \$18.

3. If $\frac{3}{8}$ of an acre be worth \$12, what is 1 acre worth?
Ans. \$32.

Questions. 20. How much greater than the dividend is the quotient of a whole number divided by a fraction? 21. How do you divide a whole number by a fraction?

4. If $\frac{3}{4}$ of the number of rows in a corn-field be 30, what is the whole number? *Answer*, 40.

5. Divide 20 by $\frac{1}{2}$—$\frac{1}{4}$—$\frac{1}{5}$. *Ans.*, 40, 80, 100.

6. If a pound of tea cost $\frac{3}{5}$ of a dollar, how many lbs. may be bought for $60? *Ans.*, 100 lbs.

7. In what time can a man build 7 rods of wall, if he build $\frac{21}{52}$ of a rod in an hour? *Ans.*, 17⅓ hours.

8. At $\frac{6}{7}$ of a dollar for building one rod of stone wall, how many may be built for $69? *Ans.*, 80½.

9. At $3¾ per yard, how many yards may be bought for $80?

10. If 1⅓ bushel of wheat sow an acre of land, how many acres will 12 bushels sow? *Ans.*, 9 acres.

11. How many times is $\frac{7}{8}$ contained in 56? *Ans.*, 64.

12. How many times is $\frac{24}{5}$ contained in 21? *Ans.* 4⅜.

Multiplication and Division of Whole Numbers by Fractions.

MULTIPLICATION.	DIVISION.
1. If a man can earn $16 in a month, how much can he earn in ⅜ of a month?	2. If a man earn $6 in ⅜ of a month, how much can he earn in a month?
3. If a man lay up $84 in a year, how much would he lay up in $\frac{5}{7}$ of a year?	4. If a man in $\frac{5}{7}$ of a year lay up $60, how much would he lay up in a year?
5. If the price of a horse be $75, what would be the price of a horse worth $\frac{4}{12}$ as much?	6. If $\frac{4}{12}$ of the value of a horse be $25, what is the whole value?
7. If a house be worth $672, how much is $\frac{9}{16}$ worth?	8. If $\frac{9}{16}$ of a house be worth $378, what is the whole worth?
9. If a farm be worth $840, what is ⅝ of it worth?	10. If ⅝ of a farm be worth $525, what is the whole worth?
11. Multiply 156 by ¾.	12. Divide 117 by ¾.
13. A man has 360 apple trees in two orchards; in the smaller, there is ⅓ of the whole. How many were there in the smaller?	14. 40 is ⅓ of what number?
15. A ship and cargo are valued at $100,000, the ship at $\frac{2}{40}$ of the whole. What is the value of the ship?	16. If $\frac{2}{40}$ of a ship and cargo are valued at $5,000, what is the worth of the whole?

Multiplication of Fractions by Fractions.

It has been shown in a preceding section, that, to multiply a whole number by a fraction, we multiply by the numerator and divide by the denominator; and also, that a fraction is multiplied, either by multiplying its numerator, or dividing its denominator, and divided by multiplying its denominator, or dividing its numerator.

If a bushel of corn be worth $\frac{2}{3}$ of a dollar, how much is $\frac{1}{2}$ of a bushel worth?

Were the cost of 2 bushels required, we should multiply the price by the quantity; but as the quantity is less than 1 bushel, we multiply by the part taken—(see p. 95.) The question then is, how much is $\frac{1}{2}$ of $\frac{2}{3}$?

If the numerators be multiplied together, and also the denominators, we have the answer—Thus, $\frac{1}{2}\times\frac{2}{3}=\frac{2}{6}=\frac{1}{3}$; or, by cancelling.

Operation.

$$\begin{array}{r|l} \cancel{2} & 1 \\ 3 & \cancel{2}=\frac{1}{3} \textit{ Ans.} \\ \hline \end{array}$$

In the first example, the price of a bushel is divided by 2, the denominator of the dividing fraction, and then multiplied by the numerator of the dividing fraction. In the second example, the process of multiplication and division is performed at once by cancelling equals on each side of the line. This process is, in fact, the same as multiplying a whole number by a fraction.

To multiply a fraction by a fraction we have this Rule.

Multiply the numerators together for a new numerator, and their denominators for a new denominator.

EXAMPLES.

1. A man owning $\frac{2}{4}$ of a ship, sold $\frac{3}{5}$ of his share—$\frac{2}{5}$ of $\frac{3}{4}$; how much?

Operation.

$$\begin{array}{r|l} & \cancel{2} \\ 5 & 3 \\ 2\ \cancel{4} & \\ \hline 10 & 3=\frac{3}{10} \textit{ Ans.} \end{array}$$

As the numerators of the fractions are to be multiplied together for a new numerator, or dividend, they are placed on the right of the line, and the denominators, which are to be multiplied for a new denominator, or divisor, are placed on the left of the line. The numbers are cancelled and multiplied as in preceding examples.

2. What is the product of $\frac{7}{9}\times\frac{6}{7}$; of $\frac{7}{12}\times\frac{3}{9}$?

Answer, $\frac{2}{3}$, $\frac{7}{36}$.

QUESTIONS. 1. How do you multiply a fraction by a fraction? 2. Why, in example 1st, are the numerators of the fractions placed on the right of the line, and the denominators on the left?

3. Multiply $\frac{7}{8}$ by $\frac{9}{2}$—$\frac{6}{7}$ by $\frac{8}{9}$.

4. A boy having $\frac{1}{2}$ of a dollar gave $\frac{1}{4}$ of it for toys, what did the toys cost him? *Ans.*, $\frac{1}{8}$ of a dollar.

5. At $\frac{3}{8}$ of a dollar per yard, what will $\frac{8}{9}$ of a yard cost? *Answer*, $\frac{3}{9}$ of a dollar.

6. At $\frac{3}{4}$ of a dollar per pound, what will $\frac{7}{8}$ of a pound of tea cost? *Answer*, $\frac{21}{32}$ of a dollar.

7. At $\frac{1}{6}$ of a dollar a pound, what will $\frac{6}{7}$ of a pound of coffee cost? *Answer*, $\frac{1}{7}$ of a dollar.

8. At $2\frac{1}{4}$ dollars per bushel, what will $6\frac{3}{27}$ bushels of wheat cost? *Answer*, \$$13\frac{3}{4}$.

9. If a house lot be worth $100\frac{5}{75}$ dollars, what is $\frac{1}{25}$ of the lot worth? *Answer*, \$$4\frac{1}{375}$.

10. If a flock of sheep be worth $75\frac{1}{5}$ dollars, what is $\frac{1}{4}$ of the flock worth? *Answer*, \$$18\frac{4}{5}$.

Division of Fractions by Fractions.

1. If a bushel of corn cost $\frac{2}{3}$ of a dollar, how much may be bought for $\frac{1}{3}$ of a dollar?

If we divide $\frac{1}{3}$ of a dollar, the amount of money to be laid out, by $\frac{2}{3}$ of a dollar, the price of a bushel, we shall have the number of bushels which $\frac{1}{3}$ of a dollar will buy. By the Rule (see p. 96,) to divide a whole number by a fraction, we multiply the dividend, or whole number, by the denominator of the fraction, and divide by the numerator. The same rule is applicable in this case. Multiply the fraction to be divided by the denominator of the fraction by which you divide, and divide by the numerator, or invert the divisor, and proceed as in Multiplication. This process will be found the same as dividing a whole number by a fraction.

Operation.

$$\begin{array}{r|l} \cancel{3} & 1 \\ 2 & \cancel{3} \\ \hline 2 & 1 = \frac{1}{2}\textit{Ans.} \end{array}$$

According to the rule already given, the numerator of $\frac{1}{3}$, the dividend, is placed on the right of the line, and the numerator of $\frac{2}{3}$, the divisor, on the left—this is the same as inverting the divisor.

2. If a bushel of potatoes cost $\frac{2}{8}$ of a dollar, how many bushels may be bought for $\frac{2}{9}$ of a dollar? *Ans.*, $\frac{8}{9}$.

3. If $\frac{6}{7}$ of a bushel of apples cost $\frac{3}{5}$ of a dollar, how much will 1 bushel cost? *Answer*, $\frac{7}{10}$.

QUESTIONS. 1. How do you divide a fraction by a fraction?

4. If $\frac{9}{16}$ of a yard of cloth cost $\frac{11}{12}$ of a dollar, how much is it per yard? *Answer*, $1\frac{17}{27}$ dolls.

5. How many bushels of rye at $\frac{8}{9}$ of a dollar per bushel may be bought for $\frac{2}{3}$ of a dollar? *Ans.*, $\frac{3}{4}$ of a bu.

6. If $\frac{2}{21}$ of a ton of hay cost $\frac{7}{8}$ of a dollar, what does it cost per ton? *Answer*, \$$9\frac{3}{16}$.

7. If $4\frac{1}{2}$ pounds of tea cost $3\frac{1}{4}$ dollars, what is it per lb.? *Answer*, $\frac{13}{18}$ of a dollar.

8. If $\frac{13}{18}$ of a dollar buy 1 pound of tea, how much will $3\frac{1}{4}$ dollars buy? *Answer*, $4\frac{1}{2}$ pounds.

9. At $1\frac{3}{7}$ dollars per yard, how much carpeting can be bought for $15\frac{5}{7}$ dollars? *Answer*, 11 yards.

10. Divide $17\frac{1}{2}$ by $7\frac{1}{2}$—$18\frac{3}{4}$ by $\frac{25}{75}$. *Ans.*, $2\frac{1}{3}$—$56\frac{1}{4}$.

Multiplication and Division of Fractions.

MULTIPLICATION.	DIVISION.
1. A man owned $\frac{7}{8}$ of a house, sold $\frac{5}{6}$ of his share, what part of the house did he sell?	2 A man sold $\frac{35}{48}$ of a house, which was $\frac{5}{6}$ of his share. What part of the house did he own?
3. If a bushel of salt cost $\frac{19}{21}$ of a dollar, what will $\frac{1}{5}$ of a bushel cost?	4. If $\frac{1}{5}$ of a bushel of salt cost $\frac{19}{105}$ of a dollar, what does it cost per bushel?
5. If a peck of coal cost $\frac{5}{61}$ of a dollar, what will $\frac{3}{4}$ of a peck cost?	7. If $\frac{3}{4}$ of a peck of coal cost $\frac{15}{244}$ of a dollar, what will one peck cost?
7. If 1 cord of wood cost $\frac{22}{5}$ of a dollar, how much will $\frac{7}{9}$ of a cord cost?	8. If $\frac{7}{9}$ of a cord of wood cost $3\frac{19}{45}$ dollars, how much is it per cord?
9. If 1 foot of hammered stone cost $\frac{15}{98}$ of a dollar, what will $\frac{2}{15}$ of a foot cost?	10. If $\frac{2}{15}$ of a foot of hammered stone cost $\frac{1}{49}$ of a dollar, what will one foot cost?

The simple rule may now be repeated for solving any question which may arise in Multiplication and Division of Fractions by Whole numbers—Multiplication and Division of Whole Numbers by Fractions—Multiplication and Division of Fractions by Fractions.

RULE.

Place all those numbers which are to be multiplied together for a numerator, or dividend, on the right of the perpendicular line, and those numbers which are to be multiplied together for a denominator, or divisor, on the left of the line, and proceed to cancel as before directed.

PROMISCUOUS EXAMPLES.

1. A man owning $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{7}{8}$ of a ship, sold $\frac{3}{7}$ of $\frac{8}{3}$ of $\frac{2}{3}$ of his share. What part of his share did he sell? *Answer*, $\frac{2}{15}$.

Thus—

$$\begin{array}{r|l} \cancel{2} & 1 \\ \cancel{3} & \cancel{2} \\ \cancel{4} & \cancel{3} \\ 5 & \cancel{4} \\ \cancel{8} & \cancel{7} \\ \cancel{7} & \cancel{3} \\ \cancel{3} & \cancel{8} \\ 3 & 2 \\ \hline 15 & 2 = \frac{2}{15}\ \textit{Ans.} \end{array}$$

Fractions connected by the word *of* are called compound fractions. They are reduced to simple fractions, by multiplying all the numerators together for a new numerator, and all the denominators for a new denominator. By cancelling, the process of multiplying and reducing the fraction, is performed at once.

2. Reduce $\frac{7}{8}$ of $\frac{3}{4}$ of $\frac{5}{7}$ of $\frac{2}{3}$ of $\frac{5}{6}$ of $\frac{8}{7}$ to a simple fraction. *Answer*, $\frac{25}{84}$.

3. A man owning $\frac{1}{2}$ of $\frac{5}{6}$ of $\frac{7}{9}$ of $\frac{9}{10}$ of a factory, sold $\frac{3}{4}$ of $\frac{3}{5}$ of $\frac{2}{3}$ of his share. What part of the factory did he sell? *Answer*, $\frac{7}{80}$.

4. What simple fraction is equivalent to $\frac{7}{8}$ of $\frac{22}{3}$ of $\frac{71}{8}$ of $\frac{5}{7}$ of $\frac{4}{18}$ of $\frac{3}{5}$ of 9, of $\frac{1}{36}$ of 18, of $\frac{4}{11}$ of 2? *Ans.* $17\frac{3}{4}$.

5. Multiply $\frac{3}{15}$ by 4.

6. Multiply $\frac{1}{3}$ by $\frac{1}{6}$.

7. Multiply $\frac{1}{2}$ by $\frac{1}{4}$.

8. Divide $4\frac{1}{2}$ by $3\frac{1}{2}$.

9. Divide $\frac{1}{4}$ of $\frac{4}{5}$ by $\frac{5}{8}$ of $\frac{8}{9}$ of 9. *Ans.*, $\frac{1}{25}$.

10. Divide $\frac{1}{2}$ of 9 by $\frac{1}{3}$ of 7. *Ans.*, $1\frac{13}{14}$.

11. Divide $\frac{1}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{7}{8}$ of 18 by $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{7}{8}$ of 12, multiply by $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{5}$ of 2, divide by $\frac{6}{7}$ of $\frac{7}{8}$ of $\frac{1}{2}$ of 6. *Answer*, $\frac{1}{15}$.

12. A man owns $\frac{1}{2}$ of a farm, sells $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{3}{7}$ of $\frac{7}{8}$ of his half, what part of the farm does he sell? *Ans.* $\frac{1}{64}$.

13. Multiply 12 by $\frac{1}{2}$ of 3, divide by $\frac{1}{4}$ of 1, multiply by $\frac{1}{2}$ of 6, divide by $\frac{6}{7}$ of 14, multiply by $\frac{1}{2}$ of 18, divide by $\frac{2}{3}$ of 27. *Answer*, 9.

Addition of Fractions.

1. What is the amount of $\frac{1}{8}+\frac{2}{8}$. If we add the nume-

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QUESTIONS. 1. What is the rule for the addition of fractions? 2. What is the first method given for reducing fractions of different denominators to fractions having a common denominator?

rators, and under their sum write the denominator, we have $\frac{3}{8}$, the answer. We have, then, this **Rule** for adding fractions. Add together their numerators, and, under the sum, write the denominator.

2. What is the amount of $\frac{1}{10}+\frac{2}{10}+\frac{3}{10}+\frac{5}{10}+\frac{7}{10}$?
Answer, $\frac{18}{10}=1\frac{8}{10}$.

3. A man gave away at one time $\frac{4}{8}$ of a bushel of corn, at another $\frac{3}{8}$, at another $\frac{6}{8}$, at another $\frac{7}{8}$, how many bushels did he give away in all? *Answer*, $\frac{20}{8}=2\frac{1}{2}$.

4. What is the amount of $\frac{2}{3}$ and $\frac{1}{6}$? We have before seen, that, to multiply or divide the terms of a fraction by any one figure, does not alter its value; therefore, if we multiply both the numerator and denominator of $\frac{2}{3}$ by 2, we have $\frac{4}{6}$, which may be added to $\frac{1}{6}+\frac{4}{6}=\frac{5}{6}$.

5. What is the amount of $\frac{1}{6}+\frac{1}{12}+\frac{1}{24}$?

If we multiply the terms of the fraction $\frac{1}{12}$ by 2, we have $\frac{2}{24}=\frac{1}{12}$, and if we multiply the terms of $\frac{1}{6}$ by 4, we have $\frac{4}{24}=\frac{1}{6}$. We can now add the fractions, $\frac{1}{24}+\frac{2}{24}+\frac{4}{24}=\frac{7}{24}$, *Answer*.

6. What is the amount of $\frac{1}{6}$, $\frac{4}{12}$, $\frac{8}{24}$? If we divide the terms of $\frac{4}{12}$ by 2, we have $\frac{2}{6}$, and the terms of $\frac{8}{24}$ by 4, we have $\frac{2}{6}$, which when added, $\frac{1}{6}+\frac{2}{6}+\frac{2}{6}=\frac{5}{6}$ *Answer*.

When fractions can be reduced to a common denominator by multiplying or dividing the terms of one or more of the fractions, the preceding mode is often convenient, but, the following is the more general **Rule**.

Multiply all the denominators together for a new denominator, and each numerator by every denominator, except its own, for a new numerator.

1. Reduce $\frac{2}{3}$, $\frac{1}{4}$ and $\frac{2}{5}$ to fractions of equal value, having a common denominator.

Operation.

denominators—$3\times4\times5=60$ com. denominator.
1st numerator—$2\times4\times5=40$ new numerator.
2d " $1\times3\times5=15$
3d " $2\times3\times4=24$

The new fractions are $\frac{40}{60}$, $\frac{15}{60}$, $\frac{24}{60}$. In this example, we first multiply all the denominators together, and obtain 60 for a new denominator. It is now necessary that each numerator should be multiplied into the same numbers by which its denominator has been multiplied, that the value of the fraction may be retained. Taking the first fraction $\frac{2}{3}$; the denominator 3 has been multiplied into 4 and 5, the

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Questions. 3. What is the general rule for finding a common denominator?

denominators of the other fractions; if, therefore, we multiply 2, the numerator, into the same numbers, 4 and 5, the value of the fraction will remain the same; since $\frac{40}{60}=\frac{2}{3}$. Again, we multiply 1, the second numerator, into 3 and 5, by which its denominator has been multiplied, and obtain 15, a new numerator. Lastly, we multiply 2, the third numerator, into 3 and 4, by which its denominator has been multiplied, and obtain 24, a new numerator. The value of the fractions remains unaltered, since $\frac{40}{60}=\frac{2}{3}$, $\frac{15}{60}=\frac{1}{4}$, and $\frac{24}{60}=\frac{2}{5}$.

EXAMPLES.

1. Reduce $\frac{1}{4}$, $\frac{1}{6}$, $\frac{2}{7}$ and $\frac{3}{8}$ to fractions having a common denominator. *Ans.*, $\frac{336}{1344}$, $\frac{224}{1344}$, $\frac{384}{1344}$, $\frac{504}{1344}$.

2. Reduce $\frac{2}{9}$, $\frac{5}{12}$, $\frac{7}{14}$ and $\frac{2}{11}$ to fractions having a common denominator. *Ans.*, $\frac{3396}{16632}$, $\frac{6930}{16632}$, $\frac{8316}{16632}$, $\frac{3024}{16632}$.

3. Add together $\frac{5}{6}$, $\frac{7}{8}$, $\frac{3}{4}$ and $\frac{2}{5}$. *Ans.*, $\frac{2941}{960}=3\frac{1}{16}$.

Note. Reduce the fractions to a common denominator, and add the numerators.

4. Add $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{3}{7}$, $\frac{7}{8}$. *Ans.*, $2\frac{31}{56}$.

5. Add together $\frac{6}{7}$ of $\frac{8}{9}$ and $\frac{9}{11}$ of $\frac{3}{13}$. *Ans.*, $\frac{2855}{3003}$.

Note. Compound fractions must be reduced to simple fractions.

6. Add $\frac{1}{3}$ of 96 and $\frac{7}{8}$ of $14\frac{1}{2}$ together. *Ans.*, $44\frac{11}{16}$.

7. Add together $\frac{1}{2}$ of $\frac{7}{8}$ and $\frac{2}{3}$ of $\frac{19}{26}$. *Ans.*, $1\frac{17}{240}$.

8. Add together 6 and $\frac{7}{8}$ of $\frac{9}{10}$ and $\frac{4}{7}$ of $\frac{1}{2}$ and $7\frac{1}{2}$. *Answer*, $14\frac{321}{560}$.

Note. Mixed Numbers may be reduced to Improper Fractions, or the fractional parts may be reduced to a common denominator, and added as in the foregoing examples. If their sum amount to an integer add it to the whole numbers.

9. Add together $14\frac{3}{4}$ and $16\frac{2}{3}$.

Operation.

$14\frac{9}{12}$ $\frac{3}{4}$, $\frac{2}{3}=\frac{9}{12}+\frac{8}{12}=\frac{17}{12}=1\frac{5}{12}$.
$16\frac{8}{12}$ We find the common denominator to
——— be 12, and the new numerators to be 9
$31\frac{5}{12}$ *Answer.* and 8, which when added are $\frac{17}{12}=1\frac{5}{12}$.

Write the $\frac{5}{12}$ under the fractions, and carry 1 to the whole numbers.

10. Add together $17\frac{3}{4}$, $18\frac{1}{2}$, $19\frac{3}{5}$. *Ans.* $55\frac{17}{20}$.

11. A grocer sold the following parcels of sugar, viz: $16\frac{1}{2}$ lbs., $19\frac{1}{4}$, $13\frac{3}{4}$, $20\frac{1}{8}$, $25\frac{1}{16}$, $30\frac{5}{6}$, and $11\frac{1}{4}$ lbs. How many pounds did he sell in all? *Ans.*, $136\frac{37}{48}$.

Subtraction of Fractions.

Rule. Prepare the fractions as in Addition, and subtract the less numerator from the greater, and under the difference write the denominator.

EXAMPLES.

1. From $\frac{7}{8}$ take $\frac{3}{4}$. *Answer*, $\frac{1}{8}$.
2. From $\frac{3}{11}$ take $\frac{1}{22}$. *Answer*, $\frac{5}{22}$.
3. From $\frac{17}{21}$ take $\frac{9}{14}$. *Answer*, $\frac{1}{6}$.
4. From $\frac{209}{216}$ take $\frac{7}{144}$. *Ans.*, $\frac{397}{432}$.
5. From $6\frac{5}{7}$ take $\frac{6}{7}$. *Ans.*, $5\frac{6}{7}$.
6. From $\frac{5}{7}$ of $\frac{3}{4}$ take $\frac{3}{5}$ of $\frac{2}{3}$. *Ans.* $\frac{19}{140}$.
7. Add together $\frac{4}{5}$ and $\frac{5}{8}$, and from their sum subtract $\frac{1}{2}$ of $\frac{9}{11}$. *Ans.*, $\frac{43}{110}$.
8. A owns $\frac{2}{3}$ of $\frac{3}{4}$ of a vessel, B owns $\frac{3}{8}$ of $\frac{4}{5}$—how much greater is A's share than B's? *Ans.*, $\frac{1}{5}$.
9. Subtract $13\frac{3}{4}$ from $15\frac{2}{3}$.

$15\frac{8}{12}$ $\frac{3}{4}-\frac{2}{3}=\frac{9}{12}, \frac{8}{12}$.
$13\frac{9}{12}$

$1\frac{11}{12}$ *Answer.*

Having reduced the fractions to a common denominator, and found new numerators as in Addition, we have $\frac{9}{12}$ to be taken from $\frac{8}{12}$; we therefore borrow the integer, and say $\frac{9}{12}$ from $\frac{12}{12}$ and add 3, the remainder, to 8, the numerator of the subtrahend. $3+8=\frac{11}{12}$, which we write under the fractions and carry 1 to 13, the whole number, which makes 14—and 14 from 15 and 1 remains—the answer, then, is $1\frac{11}{12}$.

10. A man bought a horse for $\frac{1}{2}$ of $\frac{2}{3}$ of \$150, and sold him for $\frac{1}{4}$ of $\frac{6}{7}$ of $\frac{7}{1}$ of \$60; did he gain or lose, and how much? *Ans.*, gain \$40.

To find the least Common Multiple.

The common denominator found by the preceding rule, is a common multiple of the denominators of the given fraction; for every product must be divisible by all its factors; but it was not the *least* common multiple.

1. What is the least common multiple of 4, 6, 8, 10?

$4\times6\times8\times10=1920$.

Operation.

2)4, 6, 8, 10

2)2, 3, 4, 5

1, 3, 2, $5\times2\times2=120$.

1920 is evidently a common multiple of 4, 6, 8 and 10, because they are its factors; but it is not the *least* common multiple. We find, also, that each of these numbers is a multiple of 2, because 2

Questions. 1. What is the rule for the subtraction of fractions?

is a factor in each. Dividing by 2, we find the other factors, which are 2, 3, 4 and 5. Again: As the quotient, 4, is a multiple of 2, we may substitute for it, 2, one of its factors; and as we employ the other factor for a divisor, we erase the other quotient, 2. We now have 3, 2 and 5, undivided numbers, which are found to be factors of the dividends 6, 8 and 10; the other factors are the divisors. If we multiply 3, 2 and 5 separately into the other factor or factors, the product will be each respective dividend. Thus the divisors 2 and 2 are the factors of 4, the first dividend $2\times2=4$. If 4 be divisible by 4, then 2, 3, and 5 times 4 must be divisible by 4; also 3, the first undivided number, is a factor of 6, the second dividend; and 2, the first divisor, is the other factor, $3\times2=6$; if 6 be divisible by 6, then 2 and 5 times 6 must be divisible by 6. The same may be said of 2 and 5, the other undivided numbers. Hence it appears, that the product of the continued multiplication of the remainders and divisors is divisible by the several dividends; and by examining the operation, it will be found to be the *least* number which can be divisible by them. Therefore, to find the least common multiple of two or more numbers, we have the following

RULE.

Divide by any number which will divide two or more of the given numbers without a remainder. Bring down the quotients with the undivided numbers on a line under the given numbers. Continue to divide until no two numbers are left capable of being divided by any number greater than 1. The product of the continued multiplication of the divisors and undivided numbers, will be the least common multiple required.

EXAMPLES.

2. What is the least common multiple of 3, 4, 9 and 12? *Answer*, 36.

3. What is the least number which can be divided by 7, 8, 10 and 12, without a remainder? *Answer*, 840.

4. What is the least common multiple of 7, 14, 28, 35? *Answer*, 140.

5. What is the least number which can be divided by the nine digits without a remainder? *Ans.*, 2520.

QUESTIONS. 1. How is the least common multiple of two or more numbers found?

1. Reduce $\frac{3}{4}$, $\frac{2}{5}$, $\frac{5}{6}$ to equivalent fractions having the least common denominator.

$$\begin{array}{l} 2)\underline{4,\quad 5,\quad 6} \\ \quad 2\times5\times3\times2=60 \end{array}$$

60 being a new denominator for each of the given fractions, we must find new numerators which shall bear the same ratio to 60 that each of the given numerators does to its own denominator. If we take from 60, the common denominator, the value of each fraction, for new numerators to the common denominator, we shall then have fractions of the same value of the given fractions. This is done by dividing 60 by the denominator of the fraction, and multiplying the quotient by the numerator, (see p. 96) $\frac{60}{4}=15\times3=45$, and $\frac{45}{60}=\frac{3}{4}$. The same is true of the other fractions, $\frac{2}{5}$ and $\frac{5}{6}$.

We have, then, this Rule for reducing fractions of different denominations to equivalent fractions having the least common denominator.

Find the least common multiple of all the denominators, for a common denominator. Divide the common denominator by the denominators of each of the given fractions, and multiply the quotients by the numerators of the given fractions, and the products will be the new numerators required.

2. What is the least common denominator of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{5}$, and $\frac{2}{8}$?

Operation.

$$\begin{array}{l} 4)\underline{2,\ 4,\ 5,\ 8} \\ 2)\underline{2,\ 1,\ 5,\ 2} \\ \quad 1,\ 1,\ 5,\ 1 \qquad 5\times2\times4=40 \text{ least denominator.} \end{array}$$

$$\left.\begin{array}{l} 40\div2=20\times1=20 \\ 40\div4=10\times1=10 \\ 40\div5=\ 8\times3=24 \\ 40\div8=\ 5\times2=10 \end{array}\right\}$$

Then $\frac{20}{40}$, $\frac{10}{40}$, $\frac{24}{40}$, $\frac{10}{40}$, *Answer.*]

3. Reduce $\frac{8}{9}$, $\frac{9}{11}$, $\frac{7}{12}$, to fractions having the least common denominator. *Answer*, $\frac{352}{396}$, $\frac{324}{396}$, $\frac{231}{396}$.

4. Reduce $\frac{3}{7}$, $\frac{2}{8}$, $\frac{1}{9}$ and $\frac{11}{12}$, to fractions having the least common denominator. *Answer*, $\frac{216}{504}$, $\frac{126}{504}$, $\frac{56}{504}$, $\frac{462}{504}$.

5. A merchant buys 5 pieces of cloth. The first contains $40\frac{3}{4}$ yards; the second, $27\frac{1}{2}$; the third, $34\frac{7}{8}$; the fourth $43\frac{3}{16}$; and the fifth $39\frac{1}{4}$ yards. How many were there in the whole? *Answer*, $185\frac{9}{16}$.

Questions. 2. How are fractions of different denominators reduced to equivalent fractions, having the same denominator?

6. Which is the greater fraction $\frac{11}{16}$, or $\frac{13}{18}$?
Answer, $\frac{13}{18}$ is greater by $\frac{5}{144}$.

REDUCTION OF FRACTIONS.

1. Reduce $\frac{1}{288}$ of a pound to the fraction of a penny.

We have seen that integers of a higher denomination are brought into integers of lower, by multiplication; (see p. 65,) and also that fractions are multiplied either by multiplying the numerator, or dividing the denominator. (See p. 92.) Pounds are reduced to shillings by multiplying by 20, and shillings to pence by multiplying by 12. *Therefore, to reduce* $\frac{1}{288}$ *of a pound to the fraction of a penny, multiply the fraction by* 20 *and* 12, *thus:*

$\frac{1}{288}\times20\times12=\frac{240}{288}=\frac{5}{6}$ *Answer.* Or thus:

$$\begin{array}{r|l} & 1 \\ 6\ \ \cancel{24}\ \ \cancel{288} & \cancel{20}\ \ 5 \\ & \cancel{12} \\ \hline 6 & 5=\frac{5}{6}\ \textit{Answer.} \end{array}$$

As the numerator of the fraction is to be multiplied, place it with the multipliers on the right of the line, cross both numbers, and write 24 in the place of 288; 4 is contained in 20, 5 times, and in 24, 6 times. The answer, then, is $\frac{5}{6}$, in the lowest terms of the fraction.

To change fractions of a higher denomination into fractions of a lower denomination, we have the following

RULE.

Multiply the numerator of the fraction, or divide the denominator by all the denominations between it and that denomination into which it is to be reduced, including the lower denomination.

EXAMPLES.

1. Reduce $\frac{1}{320}$ of a pound to the fraction of a penny. *Answer*, $\frac{3}{4}$.
2. Reduce $\frac{1}{3360}$£ to the fraction of a farthing. *Ans.*, $\frac{2}{7}$.
3. What part of a pound is $\frac{1}{504}$ cwt.? *Answer*, $\frac{2}{9}$.
4. Reduce $\frac{1}{24}$ of a yard to the fraction of a nail. Answer, $\frac{2}{3}$.

QUESTIONS. 1. How are fractions of a higher, reduced to fractions of a lower denomination?

5. Reduce $\frac{1}{1920}$ of a pound to the fraction of a farthing.

Operation.

$$\begin{array}{r|l} & 1 \\ \cancel{1920} & \cancel{20} \\ 2 & \cancel{12} \\ & \cancel{4} \\ \hline 2 & 1=\frac{1}{2}\ \textit{Ans.} \end{array}$$

It has been shown (see p. 65) that Reduction Ascending is the reverse of Reduction Descending, and also, (p. 93) that a fraction is divided, either by dividing its numerator, or multiplying its denominator. Farthings are reduced to pence by dividing by 4, and pence to shillings by dividing by 12; shillings to pounds by dividing by 20. Therefore—*To reduce* $\frac{1}{2}$ *of a farthing to the fraction of a pound, divide the fraction by* 4, 12 *and* 20.

1. What part of a pound is $\frac{1}{2}$ of a farthing?

Operation.

$\frac{1}{2}\times4\times12\times20=\frac{1}{1920}$. : or thus,

$$\begin{array}{r|l} & 1 \\ 2 & \\ 4 & \\ 12 & \\ 20 & \\ \hline 1920 & 1=\frac{1}{1920}\ \textit{Answer.} \end{array}$$

To change fractions of lower, into fractions of higher denominations, we have, then, this

RULE.

Multiply the denominator of the fraction by all the denominations between it and that into which it is to be reduced, and write the product under the numerator of the given fraction.

2. Reduce $\frac{5}{6}$ of a penny to the fraction of a pound.

Operation.

$\frac{5}{6}\times12\times20=\frac{5}{1440}=\frac{1}{288}$: or thus,

$$\begin{array}{r|l} 6 & \cancel{5} \\ 12 & \\ 4\ \cancel{20} & \\ \hline \textit{Answer,}\ 288 & 1\ \frac{1}{288}. \end{array}$$

As the denominator of the fraction is to be multiplied for a divisor, place it with its multipliers on the left of the line. Then, by cancelling 5 on the right of the line, and 20 on the left, equal $\frac{1}{4}$, therefore cross 5 and 20, and write 4 on the left. Multiply the remaining numbers on the left together, for a divisor. We have, then, the answer in the lowest terms of the fraction, $\frac{1}{288}$.

QUESTIONS. 2. What would be your multipliers in reducing the fraction of a pound to the fraction of a penny?

3. Reduce $\frac{3}{4}$ of a penny to the fraction of a pound. *Answer*, £ $\frac{1}{320}$.

4. Reduce $\frac{2}{7}$ of a farthing to the fraction of a pound. *Answer*, $\frac{1}{3360}$.

5. What part of a cwt. is $\frac{2}{9}$ of a pound? *Ans.*, $\frac{1}{504}$.

6. Reduce $\frac{2}{3}$ of a nail to the fraction of a yard. *Answer*, $\frac{1}{24}$.

◆

REDUCTION.

DESCENDING.	ASCENDING.
1. Reduce $\frac{1}{1440}$ of a pound to the fraction of a farthing.	2. Reduce $\frac{2}{3}$ of a farthing to the fraction of a pound.
3. Reduce $\frac{1}{400}$ of a pound to the fraction of a penny.	4. Reduce $\frac{3}{5}$ of a penny to the fraction of a pound.
5. Reduce $\frac{5}{2688}$ of a guinea to the fraction of a penny.	6. Reduce $\frac{5}{8}$ of a penny to the fraction of a guinea.
7. Reduce $\frac{4}{7}$ of a guinea to the fraction of a pound.	8. What fraction of a guinea is $\frac{4}{5}$ of a pound?
$\begin{array}{r\|l} \cancel{7} & \cancel{4} \\ \text{Guinea } 1 & \cancel{28}\text{s. } 4 \\ 5\text{ s } \cancel{20} & 1£ \\ \hline 5 & 4=\frac{4}{5}\ \textit{Answer.} \end{array}$	$\begin{array}{r\|l} \cancel{5} & \cancel{4} \\ £\ 1 & \cancel{20}\text{s. } 4 \\ 7\quad \text{s. } \cancel{28} & 1\text{ guinea.} \\ \hline 7 & 4=\frac{4}{7}\ \textit{Answer.} \end{array}$
9. Reduce $\frac{1}{360}$ of a guinea to the fraction of a penny.	10. What part of a guinea is $\frac{14}{15}$ of a penny?
11. Reduce $\frac{1}{84}$ of a shilling to the fraction of a farthing.	12. What part of a shilling is $\frac{4}{7}$ of a farthing?
13. Reduce $\frac{5}{108}$ of a pound Troy to the fraction of an ounce.	14. What part of a pound Troy is $\frac{5}{9}$ of an ounce?
15. Reduce $\frac{8}{6432}$ of a hhd. of wine to the fraction of a quart.	16. What fraction of a hhd. is $\frac{21}{67}$ of a quart?
17. What fraction of a rod is $\frac{1}{7684}$ of an acre?	18. What fraction of an acre is $\frac{40}{1921}$ of a rod?
19. Reduce $\frac{1}{9216}$ of a mile to the fraction of a rod.	20. Reduce $\frac{5}{144}$ of a rod to the fraction of a mile.
21. Reduce $\frac{1}{550440}$ of a degree to the fraction of a foot.	22. Reduce $\frac{2}{3}$ of a foot to the fraction of a degree.

23. Reduce $\frac{1}{512}$ of a bushel to the fraction of a gill.

25. Reduce $\frac{1}{322256}$ of a ton to the fraction of a gill.

27. Reduce $\frac{1}{12}$ of $\frac{1}{3}$ of 4 pounds to the fraction of a penny.

29. $\frac{7}{60}$ of a pound is $\frac{1}{6}$ of what fraction of 7 guineas?

$\frac{7}{60}$ of a pound is $\frac{1}{14}$ of what fraction of 7 guineas?

$\frac{7}{60}$ of a pound is $\frac{1}{6}$ of $\frac{1}{14}$ of how many guineas?

31. Reduce $\frac{2}{4233600}$ of a week to the fraction of a second.

33. Reduce $\frac{1}{43200}$ of a year to the fraction of an hour.

24. Reduce $\frac{1}{2}$ of a gill to the fraction of a bushel.

26. Reduce $\frac{1}{4}$ of a gill to the fraction of a ton.

28. $\frac{80}{3}$ of a penny is $\frac{1}{12}$ of $\frac{1}{3}$ of how many pounds?

$\frac{80}{3}$ of 1 penny is $\frac{1}{12}$ of what fraction of 4 pounds?

$\frac{80}{3}$ of a penny is $\frac{1}{3}$ of $\frac{1}{14}$ of what fraction of 4 pounds?

30. Reduce $\frac{1}{6}$ of $\frac{1}{14}$ of 7 guinea to the fraction of a pound.

32. Reduce $\frac{2}{7}$ of a second to the fraction of a week.

34. Reduce $\frac{1}{5}$ of an hour to the fraction of a year.

NOTE. All practical questions in the several foregoing Rules, viz. Multiplication and Division of Fractions by Whole Numbers; Multiplication and Division of Whole Numbers by Fractions; Multiplication and Division of Fractions by Fractions; also, Reduction of Fractions Ascending and Descending, may be solved by one uniform mode of stating said questions, which will be given in the Rule of Three, and which supersedes the above Rules in Fractions.

The foregoing Rules have been given as profitable exercises for the student. In practice, however, the student will be found to recur to the uniform rule stated and explained in the Rule of Three, as the most simple and expeditious process, and which he will always be able readily to retain in memory.

To reduce Fractions to integers of lower denominations, and the reverse.

1. What is the value of $\frac{2}{3}$ of a pound?

$\frac{2}{3}$ of a pound reduced to the fraction of a shilling is $\frac{2}{3}\times20=\frac{40}{3}$ of a shilling,—which, reduced to a mixed

2. Reduce 13s. 4d. to the fraction of a pound.

In £1 there are 240d. : 1 penny then is $\frac{1}{240}$ of a pound, and 160d., the number of pence in 13s. : 4d. is $\frac{160}{240}$ of number (see p 89) is 13⅓s. The ⅓ of a shilling reduced to the fraction of a penny, is $\frac{1}{3}\times12=\frac{12}{3}=4$d. Hence, to reduce fractions of one denomination to integers of a lower, we have this

Rule. Multiply the numerator of the given fraction by as many of the next lower denomination as make 1 of that denomination in which the fraction is given, and divide the product by the denominator of the fraction.—If there be a remainder, proceed as before, until it is reduced to the lowest denomination. If there be still a remainder, place it at the right of the last answer.

3. What is the value of ⅝ of a shilling?

Operation.

$$\begin{array}{r|l} 2\ \cancel{8} & 5 \\ 1 & \cancel{12}\ 3 \\ \hline 2 & 15=7\frac{1}{2} \end{array}$$ *Answer.*

5. What is the value of ⅝ of a mile?

7. What is the value of ⅛ of a degree?

9. What is the value of $\frac{9}{11}$ of a ton?

a pound, or 160 times as much as 1 penny. Therefore, to reduce integers of lower, to fractions of a higher denomination, we have this

Rule. Reduce the given numbers to the lowest denomination mentioned, for a numerator, and an integer of the denomination required to the same denomination for a denominator, and they will form the fraction required.

4. Reduce 7d. 2qrs. to the fraction of a shilling.

Operation.

Integer.	d.	qrs.
1s.	7	2
12	4	
12	6)30(5	
4	48(8 *Ans.*	
48		

Or, we may reduce the farthings to the fraction of a penny, and reduce the whole to an improper fraction.

Operation by cancelling.

Thus $7\frac{1}{2}=\frac{15}{2}$.

$$\begin{array}{r|l} 2 & \cancel{15}\ 5 \\ 4\ \cancel{12} & 1 \\ \hline 8 & 5=\frac{5}{8} \end{array}$$ *An.*

6. Reduce 6 furlongs, 26 po. 11 ft. to the fraction of a mile.

8. Reduce 8 mi. 5 fur. 20 poles, to the fraction of a mile.

10. Reduce 16 cwt. 1 qr.

Questions. 1. Rule for reducing fractions to integers of lower denominations? 2. Rule for reducing integers of lower denominations to a fraction of a higher?

11. What is the value of $\frac{4}{5}$ of a month?

13. What is the value of $\frac{5}{7}$ of a pound Troy?

15. What is the value of $\frac{5}{6}$ of an acre?

17. What is the value of $\frac{7}{8}$ of a yard of cloth?

19. What is the value of $\frac{3}{4}$ of a dollar in shillings?

21. What is the value of $\frac{5}{8}$ of a ton?

23. What is the value of $\frac{7}{8}$ of a hogshead?

12 lbs. 11 oz. 10 dr. $\frac{2}{11}$ of a dr. to the fraction of a ton.

12. Reduce 3 w. 1 d. 9 h. 36 m. to the fraction of a month.

14. Reduce 8 oz. 11 pwt. 10$\frac{4}{7}$ grs. to the fraction of a pound.

16. Reduce 3 roo. 13 ro. 90 feet, 108 in. to the fraction of an acre.

18. Reduce 3 qrs. 2 na. to the fraction of a yard.

20. Reduce 4s. 6d. to the fraction of a dollar.

22. Reduce 11 cwt. 0 qr. 12 lbs. 7 oz. 1$\frac{7}{8}$ drs. to the fraction of a ton.

24. Reduce 49 gals. to the fraction of a hogshead.

Reduction of Vulgar Fractions to Decimal.

RULE. Divide the numerator by the denominator.

1. Reduce $\frac{1}{2}$ to a decimal fraction.

Operation.

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2)10
 ——
  ,5
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In this example, $\frac{1}{2}$ being a proper fraction, the numerator will not contain the denominator, but, by adding a cipher, we obtain the quotient 5, which is 5 tenths. (See the Rule for pointing off, in Division of Decimals, p. 52.) As many ciphers, therefore, as are added to the numerator, so many decimal places must be in the quotient. If, after the division, the quotient does not contain so many, supply the deficiency by prefixing ciphers.

2. Reduce $\frac{3}{4}$ to a decimal fraction. *Answer*, ,75.

3. Reduce $\frac{5}{8}$, $\frac{1}{8}$ and $\frac{3}{5}$, to decimal fractions.
Answer, ,625 ,125 ,6.

4. Reduce $\frac{7}{8}$, $\frac{3}{8}$, $\frac{3}{4}$, $\frac{1}{4}$, and $\frac{1}{2}$, to decimal fractions.
Answer, ,875 ,375 ,75 ,25 ,5.

QUESTIONS. 1. Rule for reducing a vulgar fraction to a decimal? 2. How many decimal places must there be in the quotient? 3. If the quotient does not contain a sufficient number of figures, what is to be done?

11. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ to a decimal fraction.

Operation.

$$\begin{array}{r|l} \cancel{2} & 1 \\ \cancel{3} & \cancel{2} \\ 4 & \cancel{3} \\ \hline 4 & 1,00 \\ & \overline{,25} \end{array}$$

,25 *Answer.*

12. Reduce $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{1}{8}$ divided by $\frac{3}{8}$ of $\frac{5}{9}$ of $\frac{1}{6}$ to a decimal fraction. *Answer*, 2,25.

13. Reduce $\frac{1}{25}$ to a decimal fraction. *Answer*, ,04.

14. Reduce $\frac{162}{2563}$ to a decimal fraction. *Answer*, ,632071+.

To reduce Integers of different denominations to a Decimal Fraction of a higher denomination, and the reverse.

1. Reduce 4 pence 2 farthings to the decimal of a shilling.

Operation.

$$\begin{array}{r|l} 4 & 20 \\ 12 & 4,500 \\ \hline & ,375 \end{array}$$

2 farthings is $\frac{2}{4}$ of a penny; then, by the rule for reducing vulgar fractions to decimal, we have $\frac{2}{4}$=,5 or $\frac{5}{10}$ of a penny. This, placed at the right of 4 pence, 4,5, and divided by 12, the number of pence in a shilling, or because 4 pence is $\frac{4}{12}$ of a shilling, gives ,375 of a shilling. Hence the Rule.

Place the numbers one above another, the highest denomination at the bottom. Divide the lowest denomination by as many of that as it takes to make 1 of the next higher denomination, writing the quotient at the right of the next higher denomination,

2. Reduce ,375 of a shilling to integers of lower denominations.

As this question is the reverse of the former, and as the decimal ,375, was obtained by dividing the integers, it is plain, that the integers may be obtained by multiplying the decimal by the same numbers.

Operation.

$$\begin{array}{r} ,375 \\ 12 \\ \hline 4,500 \\ 4 \\ \hline 2,000 \end{array}$$

Hence the Rule—Multiply the given decimal by as many of the next lower denomination as it takes to make one of that in which the decimal is given; observing to point off as many pla-

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Questions. 1. What is the rule for reducing integers of different denominations to a decimal fraction of a higher denomination? 2. What is the rule for the reverse? 3. Pointing off the product is the same as what?

and so proceed until the whole shall be reduced to the required decimal.

Note. Integeis of different denominations may be reduced to a decimal of a higher, by reducing the given numbers to the lowest denomination mentioned for a numerator, and the integer, to which the given numbers are to be reduced, reduced to the same denomination for a denominator, and dividing the numerator by the denominator.

3. Reduce 8s. 4d. 2qrs. to the decimal of a pound.

Operation.

$$\begin{array}{r|l} 4 & 20 \\ 12 & 4{,}500 \\ 2|0 & 8{,}3750|0 \\ \hline & {,}41875 \end{array}$$

ces in the product for decimals as there are figures in the given decimal; and so proceed through all the denominations—and the several numbers at the left of the decimal points will be the answer required.

Note. Pointing off the product is the same as dividing by the denominator of the decimal.

4. Reduce ,41875 to integers of lower denominations.

Operation.

$$\begin{array}{rr} & {,}41875 \\ & 20 \\ \hline \text{s.} & 8{,}37500 \\ & 12 \\ \hline \text{d.} & 4{,}50000 \\ & 4 \\ \hline \text{qrs.} & 2{,}00000 \end{array}$$

5. Reduce 4 oz. 4 pwts. to the decimal of a pound.

6. What is the value of ,35 of a pound Troy?

7. Reduce 4 oz. 8 drs. to the decimal of a pound.

8. What is the value of ,28125 of a pound?

9. Reduce 2 cwt. 2 qrs. to the decimal of a ton.

10. What is the value of ,125 of a ton?

11. Reduce 3 qrs. 2 na. to the decimal of a yard.

12. What is the value of ,875 of a yard?

13. Reduce 20 h. 16 m. 48 sec. to the decimal of a day.

14. What is the value of ,845 of a day?

15. Reduce 24s. 6d. to the decimal of a guinea.

16. What is the value of ,875 of a guinea?

17. Reduce 8 rds. to the decimal of an acre.

18. What is the value of ,05 of an acre?

To reduce shillings, pence and farthings to the decimal of a pound, by Inspection.

1. Reduce 7s. 8d. 2qrs. to the decimal of a pound.

One shilling is $\frac{1}{20}$ of a pound: therefore, 2 shillings is $\frac{2}{20}$ or $\frac{1}{10}$. Having therefore, any number of shillings giv-

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Questions. 1. Rule for reducing shillings, pence and farthings to the decimal of a pound, by Inspection? 2. Rule for the reverse?

en, if we take one half the even number, they will be reduced at once, to the decimal of a pound. If there is an odd shilling, it is the same as $\frac{5}{100}$ of a pound : $\frac{1}{20}$=,05. Every farthing is $\frac{1}{960}$ of a pound, which, reduced to a decimal, is a little more than $\frac{1}{1000}$, but, if we add 1 to the number when they exceed 12 and do not exceed 36, and 2 when they exceed 36, the farthings in the given pence and farthings will be so many 1000ths of a pound.

Thus—7s. 8$\frac{1}{2}$d. is reduced to the decimal of a pound.

Operation.

,3 half of the even shillings
,05 for the odd shilling
,034 farthings in 8$\frac{1}{2}$d.
,001 for excess of 12.
,385 *Answer.*

2. Find by inspection the decimal expression of 18s. 3$\frac{1}{4}$ pence, and 17s. 8$\frac{1}{2}$d. *Answer*, £,914 and £,885.

3. Reduce to a decimal by inspection the following sums and add them together, viz:—15s. 3d.; 8s. 11$\frac{1}{2}$d.; 10s. 6$\frac{1}{4}$d.; 1s. 8$\frac{1}{2}$d.; 2$\frac{3}{4}$d. *Answer*, 1,832.

Decimals may be reduced back to shillings, pence and farthings by reversing the above process. Double the left hand figure, or tenths, for the shillings; if the second figure be 5, or greater than 5, deduct 5 from it, and add 1 to the shillings. Then consider the second and third figures so many farthings: if they exceed 12 deduct 1; if they exceed 36, deduct 2.

4. Find by inspection the value of £,385.

5. Find by inspection the value of £,927.
Answer, 18s. 6d. 2qrs.

6. Find by inspection the value of £,491, and £,984.
Answer, 9s. 9d. 3qrs.—19s. 8d. 1qr.

SUPPLEMENT TO FRACTIONS.

EXAMPLES.

1. What is the sum of $\frac{3}{4}$ and $\frac{7}{8}$? *Ans.*, $\frac{13}{8}$=1$\frac{5}{8}$.

2. If a man receive $\frac{2}{3}$ of a dollar for 1 day's work, $\frac{3}{4}$ for another, and $\frac{5}{6}$ for another, how much does he receive for the three days' work? *Answer*, 2\frac{1}{4}$.

3. If I receive $\frac{5}{7}$ of a cord of bark from one man, and $\frac{6}{9}$ from another, what part of a cord do I receive from both? *Answer*, 1$\frac{8}{21}$.

4. Add $5\frac{1}{2}$, $6\frac{1}{4}$, $2\frac{1}{8}$, $1\frac{3}{4}$. *Answer*, $15\frac{5}{8}$.

5. From $\frac{7}{8}$ take $\frac{1}{2}$. *Answer*, $\frac{3}{8}$.

6. From $12\frac{1}{2}$ take $10\frac{7}{8}$. *Answer*, $1\frac{5}{8}$.

7. Bought a piece of land $49\frac{14}{15}$ acres, sold $21\frac{3}{5}$ acres, how many were there left? *Answer*, $28\frac{1}{3}$.

8. From $\frac{1}{4}$ of a day, take $\frac{9}{40}$ of a minute.
Answer, 5 h. 59 m. $\frac{31}{40}$.

9. From $\frac{7}{8}$ of $\frac{5}{6}$ of $\frac{24}{35}$ take $\frac{1}{6}$ of $\frac{3}{4}$ of $\frac{4}{5}$. *Ans.*, $\frac{2}{5}$.

10. A man had three bags of money, containing in all 450 pounds; in the first bag he had $230\frac{17}{27}$ lbs., in the second $100\frac{11}{81}$, how many were there in the third?
Answer, $119\frac{19}{81}$.

11. From $\frac{11}{12}$ of a pound take $\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{3}{7}$ of $\frac{7}{8}$ of $\frac{8}{8}$ of 7 shillings. *Answer*, £$\frac{211}{240}$.

12. If a bushel of corn cost $\frac{5}{6}$ of a dollar, what will $\frac{1}{2}$ of a bushel cost? *Answer*, $\frac{5}{12}$.

13. If $\frac{1}{2}$ of a bushel of corn cost $\frac{5}{12}$ of a dollar, how much must be paid for 1 bushel? *Answer*, $\frac{5}{6}$.

14. If $\frac{1}{19}$ of a hogshead cost £$\frac{35}{38}$, what will be the cost of $\frac{69}{70}$ of a hhd.? *Answer*, £$17\frac{1}{4}$.

15. Multiply 30 by $4\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$, and divide the product by $\frac{1}{2}$ of 8 of $\frac{7}{8}$ of $2\frac{1}{4}$. *Answer*, $6\frac{6}{7}$.

16. Divide $\frac{1}{8}$ of $\frac{2}{3}$ by $\frac{2}{3}$ of $\frac{3}{4}$. *Answer*, $\frac{2}{3}$.

17. Reduce $\frac{8}{9}$ of a pound, Avoirdupois, to the fraction of a cwt. *Answer*, $\frac{1}{126}$.

18. Multiply $\frac{7}{8}$ of a day by $\frac{1}{4}$ of a minute.
Answer, 315 m.

19. Reduce 3s. 6d. to the fraction of a pound.
Answer, $\frac{7}{40}$.

20. Reduce $\frac{1}{126}$ cwt. to the fraction of a pound, Avoirdupois. *Answer*, $\frac{8}{9}$.

21. What is the value of $\frac{15}{16}$ of a dollar?
Answer, 5s. $7\frac{1}{2}$d.

22. What is the value of $\frac{12}{17}$ of a Julian year?
Answer, 257 d. 19 h. 45 m. $52\frac{16}{17}$ sec.

23. What is the value of $\frac{9}{14}$ of a guinea?
Answer, 18s.

24. Reduce 4 cwt. 2 qrs. 12 lbs. 14 oz. $12\frac{4}{13}$ drs. to the fraction of a ton. *Answer*, $\frac{3}{13}$.

25. Reduce 16 h. 36 m. $55\frac{5}{13}$ s. to the fraction of a day?
Answer, $\frac{9}{13}$.

26. Reduce 2 qrs. 9 lbs. 10 oz. $7\frac{21}{29}$ drs. to the fraction of a cwt. *Answer*, $\frac{17}{29}$.

27. If 100 oranges cost 10$\frac{1}{3}$s., how many hundred may be bought for 105$\frac{11}{12}$s.? *Ans.*, 10$\frac{1}{4}$.

28. How much will $\frac{1}{4}$ cwt. cost at 15$\frac{3}{4}$s. per cwt? *Answer*, 3s. 11$\frac{1}{4}$d.

29. If $\frac{3}{4}$ of a yard cost 18d., what will 1 yard cost? *Answer*, 2s.

30. If $\frac{3}{7}$ of $\frac{4}{5}$ of $\frac{7}{8}$ of a ship be worth $\frac{7}{9}$ of $\frac{6}{7}$ of $\frac{9}{16}$ of the cargo, valued at 36,000 dollars, what is the value of the ship? *Answer*, $45,000.

SIMPLE PROPORTION:

OR,

RULE OF THREE.

"The ratio, or relation of two numbers, is the quotient arising from dividing one by the other.

We introduce some examples to illustrate the theory of ratios and proportions.

1. If 13 yards of cloth cost 130 dollars, what will be the price of 18 yards of the same cloth?

If we know the price of one yard of the cloth, it is plain that we can repeat this price 18 times and thus obtain the price of 18 yards. And since 13 yards cost 130 dollars, one yard must have cost the 13th part of 130 dollars; or, performing the division, we find the result 10 dollars, which multiplied by 18, gives 180 dollars as the price of 18 yards.

2. A courier, who travels always at the same rate, having gone 5 leagues in 3 hours, how many will he go in 11 hours?

He goes in 1 hour $\frac{1}{3}$ of 5 leagues, or $\frac{5}{3}$ of one league, and of course in 11 hours he will go 11 times as far, *i. e.* $\frac{5}{3}$ of a league multiplied by $11=\frac{55}{3}$, that is, 18 leagues and one mile.

3. In how many hours will the courier of the preceding question go 22 leagues?

If we know the time he would occupy in going 1 league we should have only to repeat this number 22 times, and the result would be the number of hours required. And since it requires 3 hours to go 5 leagues, it will require

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QUESTIONS. *1. What is ratio?

only $\frac{1}{5}$ of the time, *i. e.* $\frac{3}{5}$ of an hour, to go one league; this number multiplied by 22 gives $\frac{66}{5}=13$ hours and $\frac{1}{5}$, that is 13 hours and 12 minutes.

In the preceding statements, the known numbers and those required depend on each other in a manner that it will be well to examine.

To do this we may resume the first question, in which it is required to find the price of 18 yards, of which 13 cost 130 dollars.

It is plain that the price of this piece would be double, if the number of yards were double; that the price would be triple, if the number of yards were triple, and so on; also, that for half or two thirds of the piece, we should have to pay but $\frac{1}{2}$ or $\frac{2}{3}$ of the whole price.

Hence, if there be two pieces of the same cloth, the price of the second ought to contain that of the first as many times as the length of the second contains the length of the first; and this circumstance is stated in saying, that the prices are in proportion to the lengths.

The relation, or ratio of the lengths, then, is that number, whether whole or fractional, which denotes how many times one of the lengths contains the other. If the first piece had 4 yards, and the second 8, the ratio of the former to the latter would be 2. In the given example, the first piece had 13 yards and the second 18; the ratio of the former to the latter is then $\frac{18}{13}$ or $1\frac{5}{13}$.

As the prices have the same ratio to each other that the lengths have, 180 divided by 130 must give $\frac{18}{13}$, which is the case.

The four numbers 13, 18, 130, 180, written in this order, are such that the second contains the first as many times as the fourth contains the third, and thus they form what is called a proportion.

A relation is not changed by multiplying or dividing each of its terms by the same number.

To denote that there is a proportion between the numbers 13, 18, 130 and 180, they are usually written thus, 13 : 18 : : 130 : 180, which is read 13 is to 18 as 130 is to 180; that is, 13 is the same part of 18 that 130 is of 180, or the ratio of 13 to 18 is the same as that of 130 to 180.

The first term of a relation is called an antecedent, and the second a consequent. In a proportion there are two

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QUESTIONS. 2. What is the first term of a relation called,—and what the second?

antecedents and two consequents, viz :—the antecedent of the first ratio, and that of the second, the consequent of the first ratio, and that of the second. In the proportion 13 : 18 : : 130 : 180, 13 and 130 are the antecedents, and 18 and 180 the consequents.

To ascertain that there is a proportion between the four numbers 13, 18, 130 180, we must see if the fraction $\frac{18}{13}$ and $\frac{180}{130}$ are equal—and to do this, we reduce the latter to its lowest term ; and if they be equal, as is supposed by the nature of proportion, it follows that by reducing them to the same denominators, the numerators will become equal, and consequently that 18 multiplied by 130 will give the same product as 180 by 13. This is actually the case, and proves that if four numbers are in proportion, the product of the first and last, or of the two extremes, is equal to the product of the second and third, or the two means.

We see, that if four numbers were not in proportion, they would not possess the above property; for the fraction which expresses the first ratio not being equivalent to that which expresses the second, the numerator of the one will not be equal to that of the other, when they are reduced to a common denominator.

The order of the terms of a proportion may be changed, provided they be so placed that the product of the extremes be equal to that of the means. In the proportion, 13 : 18 : : 130 : 180, the following arrangement may be made :

13 : 18 : : 130 : 180
13 : 130 : : 18 : 180
180 : 130 : : 18 : 13
180 : 18 : : 130 : 13
18 : 13 : : 180 : 130
18 : 180 : : 13 : 130
130 : 13 : : 180 : 18
130 : 180 : : 13 : 18

In each one of these, the product of the extremes is formed of the same factors, and the product of the means of the same factors. The second arrangement is one of those which most frequently occurs. Since the product of the means is equal to that of the extremes, one product may be taken for the other ; and, as in dividing the product of the extremes by one extreme, we must necessarily find the other as the quotient, so in dividing by one extreme the product of the means, we shall find the other extreme. For the same reason, if we divide the product of the extremes by one of the means, we shall find the other mean.

QUESTIONS. 3. Give an example. 4. How do you ascertain whether there is a proportion between four numbers? 5. To what is the product of the extremes equal?

We can then find any one term of a proportion, when we know the other three; for the term sought must be one of the extremes or one of the means. The operation by which, when any three terms of a proportion are given, we find a fourth, is called the "Rule of Three." There must always be three terms given, two of which are of the same kind, and the other of the kind of the answer required."

Lacroix.

◆

SIMPLE PROPORTION:

OR,

THE RULE OF THREE DIRECT.

When the ratios are in the order in which the question is proposed, the proportion is *direct.* The ratio of the first to the second term, is the same as the ratio of the third to the answer, or term sought.

In Direct Proportion, more requires more, and less requires less. If 3 men can build 12 rods of wall in a given time, how many rods can 6 men build in the same time? It is evident that 6 men would perform a greater amount of labor, in a given time, than 3 men; and, generally, the *more* men the more work in the same time; and the *more time,* the more work by the same number of men—and the reverse.

If 3 men build 12 rods, then 6 men would build $\frac{6}{3}$ of 12=24 rods: Or, if 3 men build 12 rods, then 1 man would build $\frac{1}{3}$ of 12=4 rods, and 6 men 4×6=24 rods. The ratio of 3 to 12 is the same as the ratio of 6 to 24: for $\frac{12}{3}=4$, and $\frac{24}{6}=4$; also, the ratio 3 to 6 is the same as the ratio of 12 to 24, for $\frac{6}{3}=2$ and $\frac{24}{12}=2$.

By the analysis of the foregoing question, we find that the name of the answer and that number which follows "how many," are to be multiplied together, and the product divided by that number which follows "if." The same is true, in all questions in Direct Proportion. Hence the

RULE.

Place the term of demand on that number which follows HOW MANY, HOW FAR, WHAT WILL, WHAT COST, &c. *on the*

QUESTIONS. 6. Having the extremes and one of the means given, how may the other mean be found? 7. Having the means and one of the extremes given, how may the other extreme be found? 8. What is the Rule of Three? 9. How are the ratios? 10. What is meant by the order in which the question is proposed? 11. How is a question known to belong to the Rule of Three Direct? illustrate. 12. Rule for stating the question?

*right of the perpendicular line, and that which is of the same name, or which follows "*IF,*" or "*AT,*" on the left, and the remaining number, or that which is of the same kind with the answer required, on the right. Cancel, multiply and divide, as before directed.* (See p. 44.)

NOTE. When either of the terms is a compound quantity, it may be reduced to the lowest denomination mentioned, or the lowest denomination may be reduced to a fraction of the highest.

EXAMPLES.

2. If 5 yards of cloth cost $10, what will 15 yds. cost?

Operation. How many $ | 15 yards?
If yards $\cancel{5}$ | $\$\cancel{10}$ cost 2.

|30, *Answer.*

If 5 yards cost $10, then one yard would cost $2, and 15 yards would cost 15 times 2, or $30. It will be seen by this example, that the ratio of the first term to the second is the cost of 1 yard; and as the ratio of the third term to the fourth, is the same as the ratio of the first to the second—$\frac{10}{5}=2$, and $\frac{30}{15}=2$, it follows, that if we multiply the third term by the ratio of the first to the second, we shall obtain the fourth term. Again: the first term is to the third as the second is to the fourth; for $\frac{15}{5}=3$ and $\frac{30}{10}=3$, therefore, whether we multiply 10 by 3, the ratio of 5 to 15, or 15 by 2, the ratio of 5 to 10, the result is the same.

3. If 12 yards of cloth cost $48, what will 4 yds. cost? *Answer*, $16.

4. If 4 bushels of wheat cost $8, how much will 16 bushels cost? *Answer*, $32.

5. If a man earn $24 in 12 days, how much does he earn in 6 days? *Answer*, $12.

6. If 8 yards of cloth cost $12, how much will 10 yds. cost? *Answer*, $15.

7. If 10 yards of cloth cost $15, how much will 8 yds. cost? *Answer*, $12.

8. If 6 acres of land are bought for $180, for how much may 15 acres be bought? *Answer*, $450.

9. If 15 acres of land cost $450, what will 6 acres cost? *Answer*, $180.

QUESTIONS. 13. When either of the terms is a compound quantity, what is the rule? 14. Analyze question 2d? 15. Rule for mixed numbers? 16. On which side of the line is the numerator of a fraction to be placed?

10. If 18 yards of cloth cost $36, what will 20 yards cost? *Answer*, $40.

11. If 7 men be paid $$8\frac{2}{5}$ for a certain amount of labor, what ought 25 men to receive at the same rate? *Answer*, $30.

NOTE. Mixed numbers must be reduced to improper fractions, and the numerators placed on that side of the line where the whole numbers would be placed. It may be observed, that the numerator of a fraction always occupies the same side of the line which a whole number would occupy, standing in the place of the fraction.

12. If 2 horses plough $5\frac{1}{9}$ acres in a day, how many acres would 18 horses plough in the same time? *Answer*, 46 acres.

13. If $3\frac{1}{3}$ dollars will buy $6\frac{3}{4}$ yards of cloth, how many yards will $40 buy? *Answer*, 81 yards.

14. If $12\frac{3}{7}$ dolls. buy $4\frac{1}{14}$ yds. of cloth, how many yds. will $174 buy? *Answer*, 57 yards.

15. If $1\frac{4}{11}$ of a bushel of wheat cost $$2\frac{5}{8}$, how much will 60 bushels cost? *Answer*, $$115\frac{1}{2}$.

16. If $\frac{1}{4}$ of a yard cost $\frac{2}{3}$ of a dollar, how much will $\frac{7}{8}$ of a yard cost? *Answer*, $$2\frac{1}{3}$.

17. If 3 horses consume $4\frac{1}{2}$ tons of hay in 4 months, how many tons will 22 horses consume in the same time? *Answer*, 33 tons.

18. If 1 yd. of ribbon cost 8 pence, how many dollars will 72 yds. cost? *Answer*, $8.

Operation.

$$\begin{array}{r|l} \text{How many } \$ & \cancel{72}\text{ yd.} \\ \text{yd. } 1 & 8\text{ d.} \\ \text{d. } \cancel{12} & 1\text{ s.} \\ \text{s. } \cancel{6} & 1\ \$ \\ \hline & \$8\text{ Ans.} \end{array}$$

NOTE. When the answer is required in a different denomination from that given in the supposition, follow the tables from the denomination given to the denomination required. In the last example, the price of 1 yd. is 8d. The answer is required in dollars; therefore, continue the statement by saying 12 pence make 1 shilling and 6 shillings make 1 dollar, the denomination required. Then by cancelling 6 times 12 on the left cancels 72 on the right, 8 being the only number left on the right of the line, and there being no number on the left greater than 1, 8 is the answer in dollars.

QUESTIONS. 17. Repeat the note under question 18. 18. What is the ratio of each term in the supposition to each corresponding term in the demand, in question 18?

19. If 1 pint cost 10d., what will 3 hhds. cost in pounds?

Operation.

How many £	$\cancel{3}$ hhds.
hhd. 1	63 gal.
gal. 1	$\cancel{4}$ qts.
qt. 1	$\cancel{2}$ pts.
pt. 1	$\cancel{10}$ d.
d. $\cancel{12}$	1 s.
s. $\cancel{20}$	1£

Reducing hhds. to pints, see Red. Des.

Reducing pence to £ see Reduc. Asc.

£63 Ans.

In this example the first and third terms are of different denominations, and the second term is different from the answer sought; therefore follow the tables until you find the name of the answer required. Observing to commence each successive step on the left with the denomination last placed on the right, thus,—How many £ will 3 hhds. cost? - - - - - - - - - - - if 1 hhd. is 63 galls.

and 1 gal. is 4 qts.
and 1 qt. is 2 pts.
and 1 pt. is 10 pence.
and 12 d. is 1 s.
and 20 s. is 1 £.

The facility of passing from one denomination to another will be readily seen in the statement of this question; and also that the process of Reduction Descending and Ascending employed in the ordinary mode, is rendered unnecessary by cancelling—thus, 3 times 4, on the right, are equal to 12 on the left, and 2 times 10 on the right are equal to 20 on the left; 63 being the only number left greater than 1, and standing on the right it is the answer in pounds.

20. If 3 hhds. cost £63, what will 1 pint cost in pence?

Answer, 10d.

Statement.

How many pence, d.	1 pt.
pts. 2	1 qt.
qts. 4	1 gal.
gals. 63	1 hd.
hhds. 3	63£
£1	20s.
s. 1	12d.

Having made the statement, the connection between the numbers may be shown by connecting with a continued line, first, the sign of the answer on the left with the demand on the right, and the demand with a number of the same name on the left, and this again with its equal, or its value, (as 3 hhds. with £63, its price) on the right, and thus on from left to right alternately, until you come to the name of the answer on the right, then returning with the line to the sign of the answer where you commenced.

21. If 1 pint cost 10d., what will 3 hhds. cost in dollars? *Answer*, \$210.

22. If 1 na. cost 3 farthings, how many pounds will 40 yards cost? *Answer*, £2.

23. If 1 yard of cloth cost $13\frac{1}{3}$ shillings, how much will 12 ells English cost in £? *Answer*, £10.

Operation.

$13\frac{1}{3}=\frac{40}{3}$	
How many £	$\cancel{1}\cancel{2}$ Ell E.
Ell E., 1	$\cancel{5}$ Qrs.
Qrs., $\cancel{4}$	1 Yard.
Yd., 1	$\cancel{4}\cancel{0}$ s. 10
$\cancel{3}$	
Shil., $\cancel{2}\cancel{0}$	1 £
	10£ *Answer.*

24. If 12 ells English cost £10, what will 1 yd. cost? *Ans.*, $13\frac{1}{3}$s.

25. If 1 pint of rye cost 2 pence, what will 7 chaldrons cost in guineas? *Ans.*, $85\frac{1}{3}$ guineas.

26. If 7 chaldrons cost $85\frac{1}{3}$ guineas, what will 1 pint cost in pence? *Answer*, 2d.

27. How many £ will 3 tons of lead cost at 2 farthings per ounce? How many guineas? How many dollars? *Answer*, £224—160 guineas—\$$746\frac{2}{3}$.

28. What will 20 dozen pairs of gloves cost at 4s. 6d. per pair? *Answer*, £54.

NOTE.—4 shillings 6 pence may be reduced to pence= 54 pence, or thus, $4\frac{1}{2}$ shillings=$\frac{9}{2}$s.

29. How many yards of cloth may be bought for £32 10s., at 12s. 6d. per ell English? *Answer*, 65 yds.

30. If $\frac{1}{4}$ of a yard cost $\frac{5}{6}$ of a shilling, how many guineas will 42 yards cost? *Answer*, 5 guineas.

31. If 96 lbs. of red lead cost £3 12s., what is 1 pound worth? *Answer*, 9d.

32. What is the value of $1\frac{1}{4}$ cord of bark, if $4\frac{1}{2}$ cords be worth \$20, 25? *Ans.*, \$5,$62\frac{1}{2}$.

33. If $6\frac{1}{2}$ yds. cost \$3, what will $9\frac{1}{4}$ yds. cost? *Answer*, \$4, 26, 9+.

34. How many miles will a man travel in $\frac{5}{8}$ of a day, if in $\frac{1}{2}$ of a day he travel 5120 rods? *Ans.*, 20 miles.

35. If $\frac{1}{19}$ of a barrel of flour serve 3 men 1 day, how much will be sufficient to serve 402 men the same length of time? *Ans.*, $7\frac{1}{19}$ barrels.

36. A man owning $\frac{3}{5}$ of a coal mine, sold $\frac{3}{4}$ of his share for \$36000, what was the value of the mine? *Ans.*, \$80000.

37. What will 8 bales of cloth cost, each bale containing 12 pieces, and each piece 27 yds., at $54 per piece, and what will be the cost of 1 yd?

Answer, the whole $5184, and 1 yd. $2.

38. If $100 gain $6 in one year, how much will $450 gain in the same time? *Ans.*, $27.

39. If 17 tons 12 cwt. of iron cost £165, what will be the cost of 2 cwt.? *Ans.*, 18s. 9d.

40. If 112 lbs. of beef cost 18s. 8d., what is 1 lb. worth? *Ans.*, 2d.

41. If a man travel 200 miles in 15 days, what is the average distance for every 3 days? *Ans.*, 40 m.

42. If $\frac{1}{8}$ of a yard cost $\frac{1}{5}$ of a pound sterling, what will 60 yards cost in dollars? *Ans*,. $240.

43. If 30 horses consume 70 bushels of oats in 4 weeks, how many bushels will 9 horses consume in the same time? *Ans.*, 21 bushels.

44. A merchant bought a number of bales of velvet, each containing $129\frac{17}{27}$ yds., at the rate of $7 for 5 yds;—sold the same at the rate of $11 for 7 yds. and gained $200 by the bargain. How many bales were there?

Answer, 9 bales.

Operation.

Yds. in a bale. 1st statement.

$129\frac{17}{27}=\frac{3500}{27}$

$$\begin{array}{r|l} \text{How many} \quad \$ & 5 \text{ yds.} \\ \text{yds. } 7 & 11\ \$ \\ \hline 7 & 55=7\frac{6}{7}. \end{array}$$

Sold 5 yds. for $$7\frac{6}{7}$

Bought 5 yds. for $7

Gained on 5 yds. $$\frac{6}{7}$

2d statement.

$$\begin{array}{r|l} \text{Bales} & \cancel{200}\$ \\ \cancel{3}\ \$\ \cancel{6} & \cancel{7} \\ & \cancel{5}\text{ yds.} \\ \text{yds. } \cancel{3500} & \cancel{27}\ \ 9 \\ & 1\text{ bale} \\ \hline & 9\text{ bales, } \textit{Answer}. \end{array}$$

NOTE. In the above question it is necessary to make two statements. First, to find the gain per yard. We then say, on how many bales is $200 gained, if $\frac{6}{7}$ of a dollar is gained on 5 yards, and $\frac{3500}{27}$ yards make 1 bale?

The Rule of Three Inverse is when the order of the ratios is inverted. They are not, as in the Rule of Three Direct, in the order in which the question is proposed; instead of the ratio of the first to the second term being the same as the ratio of the third to the fourth, the ratio of the first to the third is the same as the ratio of the fourth to the second.

A question is known to be in Inverse Proportion, when more requires less, or less requires more.

1. If 3 men dig a trench in 12 days, how long would it take 6 men to do the same?

Six men would evidently perform the labor of 3 men in $\frac{3}{6}=\frac{1}{2}$ of the time, $\frac{1}{2}$ of $12=6$ days. Or thus, 3 men 12 days=1 man 36 days=6 men 6 days, the answer. In this example more requires less, and it is generally true, that the greater the number of men the less the time, and the reverse. As the number of men is in the inverse ratio to the number of days, it becomes necessary in solving this question by proportion, to reverse the order of the first and second terms. As the question reads $\frac{6}{3}=2$ and $\frac{6}{12}=\frac{1}{2}$ it will be seen, that the ratios are unequal, and therefore there is no proportion; but if we reverse the first and second terms thus, $\frac{3}{6}=\frac{1}{2}$, and $\frac{6}{12}=\frac{1}{2}$, we have an equality of ratios and consequently a proportion. By the analysis of the foregoing question, we find that the number which follows *if*, and the number which is of the same name with the answer, are to be multiplied together, and the product divided by that number which follows "*how many*." The same is true in all questions in the Rule of Three Inverse.

Hence the Rule—*Place the demand of the question, or that which follows* HOW MANY, HOW LONG, *&c., on the left of the line, and the first term of the supposition, or that which follows* IF, *or* AT, *with the name of the answer, on the right of the line, and proceed to cancel as before taught.*

EXAMPLES.

2. If 4 men build a wall in 20 days, in how many days could 8 men build the same wall?

For the operation, see next page.

Questions. 1. What is the Rule of Three Inverse? 2. What is the order of the ratios? 3. How is a question known to be in Inverse Proportion? Illustrate. 4. Rule for stating the question?

Operation.

$$\begin{array}{r|l} \text{In how many days?} & \cancel{4}\text{ m.} \\ \cancel{2}\text{ m. }\cancel{8} & \cancel{20}\text{ d. } 10 \\ \hline & 10\text{ days, \textit{Answer}.} \end{array}$$

If 4 men can build a wall in 20 days, 8 men would build the same in $\frac{4}{8}=\frac{1}{2}$ of the time; $\frac{1}{2}$ of 20, or 20 multiplied by the ratio of 8 to 4=10 days, the answer.

3. If 6 men mow a field in 21 days, in how many days would 9 men mow the same? *Ans.*, 14 days.

4. If it take 9 men 14 days to mow a field, how long would it take 6 men to mow the same field? *Answer*, 21 days.

5. If it take 6 men 21 days to mow a field, how many men would mow the same in 14 days? *Ans.*, 9 men.

6. If a man perform a journey in 6 days, when the days are 16 hours long, in how many days can he perform the same when the days are 12 hours long? *Ans.*, 8 days.

7. If 1 cwt. be transported 150 miles for 1 guinea, how far can 6 cwt. be carried for the same money? *Answer*, 25 miles.

8. How many yards of carpeting ½ yard in width will cover a room 30 feet long and 20 feet wide? *Answer*, 133⅓ yds.

9. What must be the length of a garden, 16 rods in breadth, to contain 2 acres? *Ans.*, 20 rods.

10. How many yards of lining ¾ yd. wide will it take to line a cloak 4½ yds. long and 1¼ yd. wide? *Ans.*, 7½.

11. If I lend a friend $200 6 months, how long ought he to lend me $1000 to repay the kindness, allowing the month to be 30 days? *Answer*, 36 days.

12. Suppose 800 men were placed in a garrison with provision sufficient to last them 2 months, how many must depart, that the provision may last them 5 months? *Answer*, 480 men.

13. A ship's company of 15 persons is supposed to have bread to last their voyage, allowing each person 8 ounces per day. They pick up a crew of 5 persons in distress, whom they permit to share their daily allowance with them; what will be the allowance of each person? *Ans.*, 6 ounces.

14. When wheat is sold at 93 cts. per bushel, the penny loaf weighs 12 ounces—what must it weigh when wheat is $1,24 per bushel? *Answer*, 9 ounces.

15. How many yards of cloth, 1½ yd. in width, are equal in measure to 30 yds. 1 ell English in width?

Answer, 25 yards.

16. How long must a board, 4½ inches in breadth, be to contain a square foot *Ans.*, 32 inches.

17. A certain building was raised in 8 months by 120 workmen; how many workmen could have done the same amount of labor in 2 months? *Ans.*, 480. men.

18. How much in length that is 16 rods in width will it take to make an acre?

Answer, 10.

19. There is a cistern having a pipe, which will empty it in 6 hours. How many pipes of the same capacity will empty it in 20 minutes?

Answer, 18 pipes.

20. If 30 men can perform a piece of work in 11 days, how many men will accomplish another piece of work 4 times as large in a fifth part of the time?

Answer, 600 men.

Thus far, Simple Proportion has been considered as Direct and Inverse, that the scholar might have a better understanding of the doctrine of ratio; but when the nature of proportion and the question proposed are well understood, the statement may be made by the following general Rule, and solved either by Proportion or by Cancelling.

General Rule by Proportion.

Make that number which is of the same kind with the answer required, the third term, and the two remaining ones the first and second, putting the greater, or less, first, according as the 3d is greater or less than the term sought; then the fourth term will be found by multiplying the second and third, and dividing by the first.

General Rule by Cancelling.

Draw a perpendicular line, and place the sign of the answer on the left, at the top of the line, and that number which is of the same kind with the answer, on the right; and as this number is greater or less than the answer sought, place the greater or less of the two remaining numbers on the right and the other on the left, and proceed in all respects as before instructed.

Examples by Proportion.

1. If 500 men consume $102\frac{54}{73}$ barrels of flour in 9 months, how many barrels will 365 men consume in the same time?

Operation.

$102\frac{54}{73}=\frac{7500}{73}$

$500 : 365 :: \frac{7500}{73}$

7500

———

182500

2555

———

5|00)27375|00

——— bar.

73)5475(75 *Answer.*

511

———

365

365

Having reduced $102\frac{54}{73}$ barrels to an improper fraction, we make it the third term, because it is the same kind with the answer required; and as 365 men would consume a less number of barrels than 500 in a given time, we make 365 the second term and 500 the first; we then multiply the second term by the numerator of the fraction, and divide the product by 500, the first term, and the quotient divided by the denominator of the fraction is the answer.

2. A merchant bartered $5\frac{8}{9}$ cwt. of sugar, at $6\frac{3}{4}$d. per lb.—for tea, at $8\frac{5}{8}$s. per lb., how much tea did he receive?

Examples by Cancelling.

1. If 500 men consume $102\frac{54}{73}$ barrels of flour in 9 months, how many barrels will 365 men consume in the same time?

Operation.

$102\frac{54}{73}=\frac{7500}{73}$

How many bls:	$\cancel{365}$ men $\cancel{5}$
Men $\cancel{500}$	
	$75\cancel{00}$ bbls.
$\cancel{73}$	
	75 bar. *Ans.*

Having reduced the mixed number, $102\frac{54}{73}$, to an improper fraction, as the answer is in barrels, we place its numerator on the right of the line, (see p. 122,) and as the answer is to be less than this number, we place 365, the less of the other two numbers, also on the right, and 500, the remaining number, on the left. Then by cancelling—the two ciphers on the right cancel the two ciphers on the left; (this is the same as dividing both sides of the line by 100) also 73 is contained in 365—5 times. Then 5 on the right cancels 5 on the left, and 75, the remaining number on the right, is the answer.

2. A merchant bartered $5\frac{8}{9}$ cwt. of sugar at $6\frac{3}{4}$d. per lb., for tea at $8\frac{5}{8}$s. per lb.—how much tea did he receive?

Operation.

$5\frac{8}{9}=\frac{53}{9}$ cwt. $\frac{53}{9}$ of $\frac{112}{1}=\frac{5936}{9}$

$6\frac{3}{4}=\frac{27}{4}$d. $\frac{69}{8}$ of $\frac{12}{1}$d.$=\frac{828}{8}$

$8\frac{5}{8}=\frac{69}{8}$s.

$\frac{27}{4}\times\frac{5936}{9}=\frac{160272}{36}$

Invert the divisor.

8 : 160720 : : 1
828 36
36
4968 160272
2484 8
29808)1282176(43$\frac{1}{69}$ *Ans.*
119232
89856
89424

12) 12) 3)
$\frac{432}{29808}\Big|\frac{36}{2484}\Big|\frac{3}{207}\Big|\frac{1}{69}$

3. If I buy $3\frac{1}{2}$ lbs. of sugar for 25 cts., what part of a ton can I buy for $600?

Operation.

lbs. ton.
$3\frac{1}{2}=\frac{7}{2}$of $\frac{1}{28}$ of $\frac{1}{4}$ of $\frac{1}{20}=\frac{7}{4480}$

2
56
4
224
20
4480

25 : 600 : $\frac{7}{4480}$

7
25)4200(168
25
170
150
200
200

7)
8$\Big|\frac{168}{4480}\Big|\frac{21}{560}\Big|\frac{3}{80}$ *Answer.*

Operation.

$5\frac{8}{9}=\frac{53}{9}$ cwt.
$6\frac{3}{4}=\frac{27}{4}$d.
$8\frac{5}{8}=\frac{69}{8}$s.

Tea.	53 sugar.
$\cancel{9}$	
cwt. 1	$\cancel{1}\cancel{1}\cancel{2}$lbs. 7×8×53=2968
lb. 1	$\cancel{2}\cancel{7}$ d. $\cancel{8}$
$\cancel{4}$	
$\cancel{4}$ d. $\cancel{1}\cancel{2}$	1s.
s. 69	8
	1 lb. tea.
69	2968=43$\frac{1}{69}$ *Answer.*

3. If I buy $3\frac{1}{2}$ lbs. of sugar for 25 cts., what part of a ton can 1 buy for $600?

Operation.

lbs.
$3\frac{1}{2}=\frac{7}{2}$ $

Ton.	$\cancel{6}\cancel{0}\cancel{0}$
cts. $\cancel{2}\cancel{5}$	$\cancel{7}$ lbs. $\cancel{1}\cancel{2}$ 3
$\cancel{2}$	
lbs. $\cancel{2}\cancel{8}$	1 qr.
qrs. 4	1 cwt.
cwt. 20	1 ton.
80	3=$\frac{3}{80}$ *Answer.*

4. If 2 lbs. of sugar cost $\frac{1}{4}$ of a dollar, what will 100 lbs. of coffee cost, if 8 lbs. of sugar are worth 5 lbs. of coffee?

Operation.

$$\begin{array}{ccccl} \textit{coff.} & \textit{sug.} & & \textit{coff.} & \\ 5 & : 8 & :: & 100 & \text{1st STAT.} \\ & & & 8 & \\ \hline & & & 5)800 & \\ \hline & & & \text{160 lbs. sugar} & \\ \textit{sug.} & \textit{sug.} & & \textit{cts.} & \\ 2 & : 160 & :: & 25 & \text{2d STAT.} \\ & & & 160 & \\ \hline & & & 1500 & \\ & & & 25 & \\ \hline & & & 2)4000 & \\ \hline & & & \$20,00 \textit{ Answer.} & \end{array}$$

4. If 2 lbs. of sugar cost $\frac{1}{4}$ of a dollar, what will 100 lbs. of coffee cost, if 8 lbs. of sugar are worth 5 lbs. of coffee?

Operation.

$$\begin{array}{r|l} \text{Whatwill\$} & \cancel{1}\cancel{0}\cancel{0} \text{ coffee } 20 \\ \text{coffee } \cancel{5} & \cancel{8} \text{ sugar.} \\ \text{sugar } \cancel{2} & 1\$ \\ \cancel{4} & \\ \hline & \$20 \textit{ Answer.} \end{array}$$

5. If $\frac{1}{3}$ lb. less by $\frac{1}{6}$ cost 13$\frac{1}{5}$d., what will 14 lbs. less by $\frac{1}{5}$ of 2 lbs. cost? *Ans.*, £4 9s. 9d.$\frac{3}{25}$.

6. A merchant failing in trade owes $6000—his property amounts to $2400. What does his creditor receive to whom he owes $500, and what does he pay on the dollar?

$$\textit{Ans.}, \left\{ \begin{array}{l} \text{Creditor,} \\ \text{on dollar,} \end{array} \right\} \begin{array}{l} \$200 \\ 40 \text{ cts.} \end{array}$$

7. Bought $\frac{1}{2}$ yd. of cloth for \$$\frac{3}{8}$, sold $\frac{1}{3}$ of $\frac{1}{7}$ of $\frac{1}{8}$ for $\frac{5}{8}$s. Did I make or lose? *Ans.*, made 87$\frac{1}{2}$ cts.

8. If 40 yds. of cloth cost $32, what will 1 ell English cost? *Ans.*, $1.

9. If $\frac{1}{6}$ of a yd. cost $\frac{1}{5}$£ sterling, what will 60 yds. cost in dollars? *Ans.*, $240.

10. A mercer bought 3$\frac{1}{2}$ pieces of silk, each piece containing 24$\frac{1}{3}$ yds. at 6s. 6d. per yd. What does the whole cost him? *Ans.*, £27 13s. 7d.

11. If $\frac{5}{8}$ of a gallon cost $\frac{5}{8}$£, what will $\frac{5}{9}$ of tun cost? *Answer*, £140.

12. If 2$\frac{1}{2}$ yds. of cloth cost 60 cts., what will 125$\frac{3}{4}$ yds. cost? *Answer*, $30,18.

13. If $\frac{8}{9}$ of a cord of wood is worth $6, what is 40 cords worth? *Ans.*, $470.

COMPOUND PROPORTION.

When a proportion is formed by the combination of two or more simple proportions, it is called Compound Proportion, or Double Rule of Three.

1. If 8 men consume 24 bushels of wheat in 5 months, how many bushels will 4 men consume in 15 months?

In this question, the number of bushels consumed depends on two circumstances—the number of men, and the time. We may consider the circumstances separately, and solve the question by two statements in the Single Rule of Three. First, the number of men. If 8 men consume 24 bushels in 5 months, how many bushels will 4 men consume in the same time?

Operation.

How many bushels.	$\cancel{4}$ men.
$\cancel{2}$ men $\cancel{8}$	$\cancel{24}$ bushels 12
	12 bushels, *Answer*.

Secondly, the time. If 4 men consume 12 bushels in 5 months, how many bushels will the same number of men consume in 15 months?

Operation.

How many bushels.	$\cancel{15}$ mo. 3
mo. $\cancel{5}$	12 bu.
	36 bu. Answer.

All questions in Compound Proportion may be solved by two or more statements in Simple Proportion, or they may be analyzed—thus: If 8 men consume 24 bushels in 5 months, 1 man would consume $\frac{1}{8}$ of $24=3$ bushels, and in 1 month $\frac{1}{5}$ of $3=\frac{3}{5}$ of 1 bushel. Then 4 men would consume 4 times $\frac{3}{5}=\frac{12}{5}$, in 1 month, and in 15 months, 15 times $\frac{12}{5}=36$ bushels, the answer.

Note.—The foregoing is, perhaps, the most natural method of solving proportional questions, but it is not always the shortest; other methods are adopted rather for the sake of brevity than correctness.

Compound Proportion teaches to solve by one state-

Questions. 1. How is Compound Proportion formed? 2. What does Compound Proportion teach? 3. What is a compound ratio?

ment, questions which would require two or more, by *Simple Proportion.*

2. If a man build 27 rods of wall in 3 days, when the days are 12 hours long, how many rods can he build in 9 days, when the days are 16 hours long?

If a man, in 3 days, build 27 rods, in 1 day he would build $\frac{1}{3}$ of $27=9$ rods. If in one day, 12 hours long, he build 9 rods, in 1 hour he would build $\frac{9}{12}$ of a rod, and in 16 hours $\frac{9}{12}\times16=\frac{144}{12}=12$ rods. If in 1 day, 16 hours long, he build 12 rods, in 9 days he would build $12\times9=108$ rods, the answer.

In this example, 3 days, 12 hours long, are equal to $12\times3=36$ hours, and 9 days, 16 hours long, are equal to $16\times9=144$ hours. We have, then, this proportion—36 h. : 144h. : : 27 rds. : 108 rods for $\frac{144}{36}=4$ and $\frac{108}{27}=4$. The ratio of the time in the supposition, to the time in the demand, is the same as the ratio of the term sought to the rods in the conditions of the question. That is, the ratio of 36 hours to 144 hours, expresses how many more rods can be built in 9 days, 16 hours long, than in 3 days, 12 hours long.

It will be perceived, that the ratio of the time in the supposition to the time in the demand, is the product of two simple ratios. It is a ratio of the ratio of days to days, and hours to hours, (a ratio produced by the multiplication of simple ratios, is called a compound ratio.) Thus—if a man in 3 days 12 hours long, build 27 rods of wall, the amount of wall built in 9 days, (the days being of equal length,) is expressed by the ratio of 3 to 9—$\frac{9}{3}=3$, that is, he could build 3 times the number of rods—$27\times3=81$ rods; but the days are 16 hours long: this circumstance, again, affects the result, and is expressed by the ratio of 12 to 16—$\frac{12}{16}=1\frac{1}{3}$; that is, the amount of labor performed in 16 hours is greater by $\frac{1}{3}$ than the labor performed in 12 hours, $\frac{1}{3}$ of 81 rds.$=27$, and $27+81=108$ rods. If we multiply these simple ratios, $3\times1\frac{1}{3}=4$, we have a compound ratio, the same as above, and $27\times4=108$ rods, the same answer. The ratio of the days to the hours may be expressed thus.—If, in 3 days, 12 hours long, 27 rods of wall are built, how many rods can be built in $\overset{\text{days.}}{\frac{9}{3}}$ of $\overset{\text{hours.}}{\frac{16}{12}}$?—$\frac{9}{3}$ of $\frac{16}{12}=\frac{144}{36}=4$, and $27\times4=108$, the answer.

From the preceding illustrations we derive the following *general* RULE :

General Rule by Proportion.

Make that number, which is of the same kind with the answer, the third term; and of the remaining numbers, take any two that are of the same kind, and place one for a first term, and the other for a second term, according to the directions in Simple Proportion; then, any other two of the same kind, and so on till all are used. Lastly, multiply the third term by the product of the second terms, and divide the result by the product of the first terms, and the quotient will be the fourth term, or answer.

EXAMPLES.

1. If a man build 27 rods of wall in 3 days, when the days are 12 hours long, how many rods can he build in 9 days, when the days are 16 hours long?

Operation.

$$\left.\begin{array}{r} 3\text{ d.} : 9\text{ d.} \\ 12\text{ h.} : 16\text{ h.} \end{array}\right\} :: 27\text{ rds.}$$

$$\begin{array}{rr} \underline{} & \underline{} \\ 36 & 144 \\ & \underline{27} \\ & 1008 \\ & \underline{288} \\ & \text{rds.} \\ 36)3888(108, & \textit{Answer.} \\ \underline{36} & \\ 288 & \\ 288 & \end{array}$$

Having placed 27 rods for the third term, we place 9 days for the sec-

General Rule by Cancelling.

Draw a perpendicular line, and place the sign of the answer on the left, and that number which is of the same kind with the answer, on the right; and of the remaining numbers, take any two of the same kind, and place one on the right and the other on the left, according to the directions in Simple Proportion; then, any other two of the same kind, and so on till all are used. Then cancel, multiply and divide, as before directed.

EXAMPLES.

1. If a man build 27 rods of wall in 3 days, when the days are 12 hours long, how many rods can he build in 9 days, when the days are 16 hours long?

Operation.

$$\begin{array}{r|l} \text{How many rods.} & \cancel{9}\text{ days }\cancel{3} \\ \text{days }\cancel{3} & \cancel{16}\text{ hours }4 \\ \cancel{3}\text{ hours }\cancel{12} & 27\text{ rods} \\ \hline & 108\text{rods, } \textit{Answer.} \end{array}$$

Having placed 27 rods, the name of the answer, on the right; and, as a greater amount of labor can be accomplished by 1 man in 9 days than in 3 days, and in 16 hours than in 12 hours, we place 9 and 16 on the right, and the remaining numbers on the left.

ond; for it is obvious, that a greater amount of labor could be accomplished in 9 days by one man, than in three days. For the same reason, 16 hours is made to occupy the second place, and 12 hours the first.

It has been shown by the foregoing illustration, that the amount of labor done in 9 days, 16 hours long, compared with 3 days, 12 hours long, is expressed by the product of the ratios of 3 to 9, and 12 to 16. Hence, by cancelling: If we divide 9 on the right by 3 on the left, and 16, on the right, by 4; and 12 on the left, by 4, we have then, 3 on each side of the line, which cancel each other, and 4 remaining on the right is the product of the ratios, as before shown: 4 times 27 rods is 108 rods—the answer.

2. If 3 men can build 360 rods of wall in 24 days, how many rods can 8 men build in 27 days?

Operation.

$$\begin{array}{lll} 3 : & 8 : : & 360 \\ 24 & 27 & \\ \hline 72 & 216 & \\ & 360 & \end{array}$$

12960
648

rds.

72)77760(1080, *Answer.*
72

576
576

Ques. 2d, by cancelling.

Operation.

$$\begin{array}{r|l} \text{How many rods?} & \cancel{8} \\ \cancel{3} & \cancel{27}\ \ 3 \\ \cancel{3}\ \ \cancel{24} & 360 \\ \hline \end{array}$$

Rods, 1080, *Answer.*

☞ *For Question 3d, by Cancelling, see next page.*

3. How many men will it take to build a wall 75 rods long, 8 feet high, 3 feet thick, in 6 days, working 9 hours

each day, if 20 men can build a wall 100 rods long, 6 feet high, 4 feet thick, in 12 days, working 12 hours each day?

Statement.

$$\left.\begin{array}{r}100\\6\\4\\6\\9\end{array}\right\}\begin{array}{c}\\:\\ \\:\\ \\ \end{array}\left.\begin{array}{r}75\\8\\3\\12\\12\end{array}\right\}\begin{array}{l}\\::\\20\text{ men.}\\::\\ \\ \end{array}$$

Operation.

$$\begin{array}{r@{\qquad}r}100 & 75\\6 & 8\\\hline 600 & 600\\4 & 3\\\hline 2400 & 1800\\6 & 12\\\hline 14400 & 21600\\9 & 12\\\hline 129600 & 259200\\ & 20\\ & \overline{}\text{ men.}\end{array}$$

$$\begin{array}{l}129600)5184000(40\,Ans\\\underline{518400}\\0\end{array}$$

Ques. 3d, by cancelling.

Operation.

$$\begin{array}{r|l}\text{How many men?} & \cancel{7}\cancel{5}\quad 5\times5=40\text{ men Ans.}\\\cancel{5}\ \cancel{1}\cancel{0}\cancel{0} & 8\\\cancel{3}\ \cancel{6} & \cancel{3}\\\cancel{4} & \cancel{1}\cancel{2}\ \ \cancel{3}\\\cancel{6} & \cancel{1}\cancel{2}\ \ \cancel{2}\\\cancel{9} & \cancel{2}\cancel{0}\text{ men.}\\\hline & 40\text{ men, Ans.}\end{array}$$

4. If $\frac{3}{4}$ of a yard of cloth, $\frac{7}{8}$ yd. wide, cost $\frac{2}{5}$ £—what is the value of $\frac{5}{8}$ yard, $1\frac{3}{4}$ yard wide of the same quality?

Operation.

$\frac{3}{4} : \frac{5}{8} ::$

$\frac{7}{8} : \frac{7}{4} :: \frac{2}{5}$£ $[=\frac{7}{16}$

$\frac{3}{4}\times\frac{7}{8}=\frac{21}{32}$ $\frac{5}{8}\times\frac{7}{4}=\frac{35}{32}\times\frac{2}{5}=\frac{70}{160}$

Inverted.

$$\frac{32}{21}\times\frac{7}{16}=\overset{8)}{\frac{224}{336}}\Big|\overset{7)}{\frac{28}{42}}\Big|\overset{2)}{\frac{4}{6}}\Big|\frac{2}{3}\ \textit{Ans.}$$

$$\begin{array}{r}16\\\hline 126\\21\\\hline 336\end{array}$$

4. If $\frac{3}{4}$ of a yard of cloth, $\frac{7}{8}$ yd. wide, cost $\frac{2}{5}$ £, what is the value of $\frac{5}{8}$ yd., $1\frac{3}{4}$ yd. wide, of the same quality?

Operation by cancelling.

$$\begin{array}{r|l}\text{How many £} & \cancel{5}\text{ yds. long.}\\\cancel{8} & \\4\!\!\!\cancel{} & \cancel{7}\text{ yds. wide.}\\3 & \cancel{4}\\\text{If yard long, Yard wide, }\cancel{7} & \cancel{8}\\\cancel{5} & 2\text{ £ cost.}\\\hline 3 & 2=\frac{2}{3}\text{ £ Answer.}\end{array}$$

5. If a man travel 240 mls. in 12 days, when the days are 12 hours long, how far can he travel in 27 days, when the days are 16 hours long?

Operation.

$$\left.\begin{array}{l} 12\text{ d.} : 27\text{ d.} \\ 12\text{ h.} : 16\text{ h.} \end{array}\right\} : : 240\text{ m.}$$

$$\begin{array}{rr} 144 & 162 \\ & 27 \\ \hline & 432 \\ & 240 \\ \hline & 17280 \\ & 864 \\ \hline \end{array}$$

$$\begin{array}{l} 144)103680(720\text{ mls. }\textit{An.} \\ \quad 1008 \\ \hline \quad\ \ 288 \\ \quad\ \ 288 \\ \hline \quad\quad\ 0 \end{array}$$

5.' If a man travel 240 miles in 12 days, when the days are 12 hours long, how far can he travel in 27 days, when the days are 16 hours long?

Operation.

$$\begin{array}{r|l} \text{How far ?} & \cancel{27}\text{ days } 9 \\ \text{Days } \cancel{12} & \cancel{16}\text{ hours } 4 \\ \cancel{4}\text{hours } \cancel{12} & \cancel{240}\text{ miles } 20 \\ \hline & 720\text{ miles, }\textit{Ans.} \end{array}$$

In *Compound Proportion*, the terms in the supposition and demand may be distinguished by *cause* and *effect*, or *producing* and *produced terms*. That which causes any thing, or produces an effect, as men, time, length, breadth, depth, &c., may be denominated a *producing term*. Thus, in the foregoing question, among the terms of supposition, one man, 12 days, 12 hours long, are the joint cause, or *producing terms*, and miles, the effect, or *produced term*. Among the terms of demand, 27 days, 16 hours long, are the joint cause, or the producing terms, and the rods required, are the effect, or the *produced term*. In all questions in Proportion, the answer required will be either *cause* or *effect* (a *producing* or *produced term.*) Hence the

RULE.

When the term required is a produced term,

Draw a perpendicular line, and place the terms of demand

QUESTIONS. 1. What is meant by *producing* and *produced* terms? 2. Rule, when the term required is a produced term?

or those numbers which follow "HOW," &c., on the right of the line, and those which follow "IF," on the left, as in the Rule of Three Direct, closing the statement, in all cases, with that number which is of the same kind with the answer sought, and proceed to cancel as before instructed.

NOTE. All questions under the foregoing head will be found to be in *Direct Proportion*.

6. If 4 students spend £19 in 3 months, how many £ will 8 students spend in 9 months? *Answer*, £114.

Operation.

	How many £ ?	8 students.	Producing terms of the demand.
Producing terms of the supposition.	Students 4	9 months.	
	Months 3	19£.	Produced term of the supposition.
		114£ *Answer*.	

7. If 7 men can reap 84 acres in 12 days, 12 hours long, how many acres can 20 men reap in 5 days, 14 hours long? *Answer*, 116⅔ acres.

8. If 8 reapers receive £3 4s. for 4 days' work, how much ought 20 reapers to receive for 15 days' work? *Answer*, £30.

9. If 3 men receive £8,9 for 19,5 days' labor, how much ought 20 men to receive for 100,25 days' labor? *Answer*, £305 0s. 8d. 1qr.

10. If 20 cwt. may be carried 80 miles for $35, how much will it cost to transport 40 cwt. 100 miles? *Answer*, $87½.

11. If the freight of 9 hhds. of sugar, each weighing 12 cwt., 20 leagues, cost £16, what must be paid for the freight of 50 tierces, each weighing 2½ cwt., 100 leagues?

NOTE. Hundred weight and distance are the producing terms, and the money received, the produced term.

12. If 2¼ yards of cloth, 1⅗ yard wide, cost £3⅗, how much will 36½ yards, 1½ yard wide, cost? *Ans.*, £54¾.

13. If 24 bushels of wheat be consumed by 8 persons in 5 months, how many persons will consume 36 bushels in 15 months?

Operation.

How many persons ?	$\cancel{36}$ bushels 4
$\cancel{3}$ bushels $\cancel{24}$	$\cancel{5}$ months.
$\cancel{3}$ men $\cancel{15}$	$\cancel{8}$ persons.
	4 persons, *Answer*.

It is evident, that if 24 bushels are consumed by 8 persons in 5 months, that 1 person would consume $\frac{1}{8}$ of $24=3$, and in 1 month $\frac{3}{5}$ of a bushel, and in 15 months $\frac{3}{5}\times 15=\frac{45}{5}=9$ bushels. Then $36\div 9=4$ persons, the answer. It will be observed, that the answer required is a *producing term;* and, also, that the product of the 36 bushels, the produced term in the demand, and 8 persons, 5 months, the producing terms in the supposition, is divided by the product of 24 bushels and 15 persons, the produced and producing terms in the supposition. Hence the RULE :

When the term required is a producing term,
Draw a perpendicular line, and place the produced term of the demand and the producing terms of the supposition on the right, and the remaining terms on the left of the line, and proceed as before.

NOTE. The above will be found to correspond with *Inverse Proportion*, and may be applied to both Simple and Compound.

14. If 6 men build a wall 20 feet long, 6 feet high and 4 feet thick, in 16 days, working 12 hours in a day, in how many days will 24 men build a wall 200 feet long, 8 feet high, and 6 feet thick, working 10 hours in a day?

Answer, 96 days.

Operation.

How many days?		$\cancel{2}\cancel{0}\cancel{0}$	Produced term of demand.
Produced term of supposition.	$\cancel{2}\cancel{0}$	8	
	$\cancel{6}$	$\cancel{6}$	
	$\cancel{4}$	$\cancel{6}$	Producing terms of supposition.
Producing terms of demand.	$\cancel{4}\ \cancel{2}\cancel{4}$	12	
	$\cancel{1}\cancel{0}$	$\cancel{1}\cancel{6}$	
		96 days, *Answer*.	

15. If 3 men in 24 days, 9 hours long, can dig 328 rods of trench, 6 feet wide and 4 feet deep, how many men will it take to dig a trench 984 rods long, 9 feet wide and 8 feet deep, in 27 days, when the days are 12 hours long?

Answer, 18 men.

16. If a man can travel 240 miles in 16 days, when the days are 14 hours long, how many days will it take him to travel 720 miles, when the days are 12 hours long?

Answer, 56 days.

17. If 98 lbs. of bread be sufficient to serve 7 men 14 days, how many days will 63 lbs. serve 21 men?

Answer, 3 days.

QUESTIONS. 3. Rule, when the term required is a *producing* term?

18. If 40 men in 15 days, 12 hours long, build a wall 200 feet long, 12 feet high and 5 feet thick, how many hours long must the day be, that 20 men, in 12 days, may build a wall 100 feet long, 10 feet high and 6 feet thick? *Answer*, 15 hours.

19. If 20 men in 12 days, 15 hours long, can build a wall 100 feet long, 10 feet high and 6 feet thick, in how many days, of 12 hours long, can 40 men build a wall 200 feet long, 12 feet high and 5 feet thick? *Answer*, 15 days.

20. If 16 compositors set 150 pages of types, each page consisting of 48 lines, and each line of 50 letters, in 8 days, 10 hours long, how many compositors will be required to set 500 pages of 72 lines each, and 45 letters in a line, in 6 days, 8 hours long? *Answer*, 120 compositors.

SUPPLEMENT

TO THE RULES OF PROPORTION.

EXAMPLES.

1. If I can hire 30 horses pastured 7 weeks for £6, how many weeks may 5 horses be pastured for £4 5s. 8$\frac{4}{7}$d.? *Answer*, 30 weeks.

2. If 8 men build 48 rods of fence in 1 day, how many rods will 24 men build in the same time. *Ans.*, 144 rods.

3. If 24 men build 144 rods of fence in 1 day, how many rods will 8 men build in the same time? *Answer*, 48 rods.

4. How many men will it require to build 144 rods of fence in 1 day, if 8 men build 48 rods in the same time? *Answer*, 24 men.

5. If 20 men build a mill in 160 days, in how many days could 25 men build the same mill? *Answer*, 128 days.

6. If 25 men build a mill in 128 days, how many men will build the same mill in 160 days? *Ans.*, 20 men?

7. How many cords of wood may be bought for £40, if 2$\frac{2}{5}$ cords cost $6? *Answer*, 53$\frac{1}{3}$ cords.

8. How many bushels of wheat may be bought for £40, if 1 bushel of wheat be worth 2 bushels of rye, and 4 bushels of rye be worth 5 bushels of corn, and 8 bushels

of corn be worth 16 bushels of oats, and 1 bushel of oats be worth 2 shillings? *Answer*, 80 bushels.

9. If 18 cords of oak wood be worth 26$\frac{2}{3}$ cords of hemlock, and a cord of hemlock 14$\frac{2}{5}$ shillings, how much will 4$\frac{1}{2}$ cords of oak cost in cents, and how many guineas?

Answer, $\left\{\begin{array}{l}\text{1600 cents.}\\ 3\frac{3}{7}\text{ guineas.}\end{array}\right.$

10. How many stoves may be bought for 672 shillings, if 8 fire-frames are worth 3$\frac{1}{3}$ stoves, and 16 fire-frames are worth 32 guineas? *Answer*, 5 stoves.

11. How much will $450 gain in a year, if 100 in the same time gain $8? *Answer*, $36.

12. A merchant owning $\frac{2}{5}$ of a vessel, sold $\frac{5}{8}$ of his share for $934. What was the value of the ship?

Answer, $3736.

13. How many guineas will 8$\frac{2}{5}$ yards of cloth cost, if $\frac{7}{8}$ of yard cost $\frac{5}{8}$ of a dollar? *Ans.*, 1$\frac{2}{7}$ guinea.

14. If $\frac{11}{13}$ of a pound of sugar cost $\frac{7}{15}$ of a shilling, what will $\frac{32}{43}$ of a pound cost? *Ans.*, 4d. 3qr. $\frac{1657}{2365}$.

15. If 2$\frac{1}{2}$ lbs. of tobacco cost 4s. 6d. how much will 180 lbs. cost in dollars? *Ans.*, $54.

16. If when wheat is 4s. 6d. per bushel the penny loaf weigh 12oz., what ought it to weigh when wheat is $,50 per bushel? *Ans.*, 18oz.

17. How many yards of cloth, $\frac{3}{4}$ yd. wide, are equal in measure to 30 yds. 1$\frac{1}{2}$ yard in width? *Ans.*, 60 yds.

18. If 40 bushels of grain will pay a debt when the price is 60 cents per bushel, how many bushels will it take when the price is $1,20? *Ans.*, 20 bushels.

19. How far may 30 cwt. be transported for $8, if 2$\frac{1}{2}$ cwt. be carried 180 miles for the same money?

Ans., 15 miles.

20. How many yards, of $\frac{3}{4}$ yd. wide, will it take to line 850 suits of clothes, each suit to contain 3$\frac{1}{2}$ yards of cloth, 1$\frac{3}{4}$ yd. in width? *Ans.*, 6941 yds., 2 qrs., 2$\frac{2}{3}$ nails.

21. If 30 horses consume 600 bushels of oats in 8 mo. how much will each horse consume per day?

Ans., 2$\frac{2}{3}$ qts.

22. A man owns $\frac{1}{2}$ of a ship, valued at $\frac{1}{4}$ of the ship and cargo—the latter worth $96,000. What is the value of $\frac{1}{4}$ of his share? *Ans.*, $4,000.

23. If 12 men consume $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of 30 bushels of wheat, in $\frac{1}{2}$ of $\frac{14}{39}$ of $\frac{6}{7}$ of $\frac{8}{15}$ of 20 months, how much

will 4 men consume in $\frac{3}{8}$ of $\frac{6}{7}$ of $\frac{16}{19}$ of $\frac{38}{39}$ of $\frac{14}{15}$ of 40 months? *Ans.*, 12 bushels.

24. If 4 men spend $\frac{3}{4}$ of $\frac{8}{9}$ of $\frac{6}{7}$ of $\frac{14}{15}$ of £30 in $\frac{7}{11}$ of $\frac{8}{13}$ of $\frac{26}{27}$ of $\frac{11}{7}$ of 9 days, how many dollars will 21 men spend in $\frac{3}{4}$ of $\frac{14}{15}$ of $\frac{6}{7}$ of $\frac{4}{12}$ of 45 days. *Ans.*, $420.

25. If a man travel 336 miles in 14 days, when the days are 18 hours long, in how many days will he travel 672 miles, the days being 12 hours long? *Ans.*, 42 days.

26. If a man travel 240 miles in 12 days, when the days are 12 hours long, in how many days will he travel 720 miles, when the days are 16 hours long? *Ans.* 27 days.

27. If a family of 9 persons spend $450 in 7 months, how much would be sufficient to maintain them 8 months, if 5 persons more were added to the family?
Ans., $800.

28. What is the value of 1 grain of gold, if 17$\frac{2}{3}$ lbs. be worth £10224? *Ans.*, 2$\frac{1}{106}$s.

EXCHANGE.

Exchange teaches to find the relative value of money in different states and countries.

When the United States were British colonies, the sterling value of the pound was the same in all the colonies; but the legislatures of the different colonies emitted bills of credit, which afterwards depreciated in their value—in some states more, and in others less.

The following table exhibits the number of shillings in a dollar in each of the states.

TABLE I.

To exchange from to	New-Eng. States and Virginia.	Penn'a, N. Jersey, Delaware and Maryland.	New-York and North Carolina.	South-Carolina and Georgia.
New England States and Virginia.	Dollar. 6s. 0d.	Add one 4th.	Add one 3d.	×7 and ÷9.
Pennsylvania, N.Jersey, Delaware &Marylnd	Subtract one 5th	Dollar 7s. 6d.	Add one 15th.	×3$\frac{1}{9}$ and ÷5.
New-York and North-Carolina.	Subtract one 4th.	Subtract one 16th.	Dollar 8s. 0d.	×7 and ÷12.
South-Carolina and Georgia.	Add 2-7ths.	×5 and ÷3$\frac{1}{9}$.	× by 12 ÷ by 7	Dollar, 4s. 8d.

Note. The value of a dollar in any State, is found either opposite to that State, or under it in the table.

As the *number* of shillings in a dollar is different in different States, the value of the dollar being the same, it follows, that the *value* of the shilling is different; and as the number of shillings in a pound is the same, the value of the pound must differ in the ratio of the shillings. That is: if 6 shillings in New-England, and 8 shillings in New-York, make one dollar, then a pound in New-York is to a pound in New-England, as 8 is to 6; $\frac{8}{6}=\frac{4}{3}$; or, £4 in N. York are equal to £3 in New-England.

The relative value of the pound in different States, may be seen by the following table.

TABLE II.

	£ £	
New-Jersey, Pennsylvania, Delaware and Maryland.	15=16	New-York and North-Carolina.
	5= 4	New-England and Virginia.
	45=48	South-Carolina and Georgia.
New-England States and Virginia.	4=5	New-Jersey, Pennsylvania, &c.
	3=4	New-York and North-Carolina.
	9=7	South-Carolina and Georgia.
New-York and N. Carolina.	16=15	New-Jersey, Pennsylvania, &c.
	4= 3	New-England and Virginia,
	12= 7	South-Carolina and Georgia.
S. Carolina and Georgia.	28=45	New-Jersey, Pennsylvania, &c.
	7=12	New-York and North-Carolina,
	7= 9	New-England and Virginia.

The following table shows the value of pounds, shillings and pence in each of the United States, according to their respective currencies.

TABLE III.

New-Jersey, Pennsylvania, Delaware and Maryland.	New-York and N. Carolina.	New-England States and Virginia.	S. Carolina and Georgia.
£ 3=$8	£ 2=$5	£ 3=$10	£ 7=$30
s. 3=40 cts.	s. 2=25 cts.	s. 3=50 cts.	s. 7=1,50cts
d. 9=10 cts.	d. 24=25 cts.	d. 18=25 cts.	d. 14= 25 cts.

Questions. 1. What does Exchange teach? 2. Why is the value of the pound different in different States? 3. Is the value of the dollar every where the same? 4. Is the value of the shilling the same?

DOMESTIC EXCHANGE.

To reduce the currency of one State to that of another.

RULE.

Draw a perpendicular line, and place the demand of the question on the right, and the supposition on the left, as in the Rule of Three Direct.

EXAMPLES.

1. What sum in Georgia is equal to £1800 New-England currency?

Operation.

How many £ Georgia	$\cancel{1}\cancel{8}\cancel{0}\cancel{0}$ £ N. E.
If N. E. £ $\cancel{9}$	7 G. 200
	1400 £ *Answer.*

In stating the question, say how many £ Georgia currency are equal to £1800 New-England, if £9 New-England are equal to £7 in Georgia. (See Table II.)

2. How many pounds in New-York currency, will £240 New-Jersey currency make? *Answer,* £256.

3. What sum in Virginia is equal to £375 16s. 9d. N. York currency? *Answer,* £281 17s. 6d. 3qrs.

4. What is the value in N. Jersey currency, of a bill of exchange for £375 10s., on a cotton dealer in Georgia?
Answer, £603 9s. 7d. 3qrs.

5. A manufacturer in Massachusetts, sends to Georgia a lot of shoes, which amount to £420 7s. What is the value in New-England currency? *Answer,* £540 9s.

6. A manufacturer in New-Jersey, consigns to his agent in Charleston, a quantity of *ready made* clothing, which, when sold, and the charges deducted, amounted to £532 11s. What is the value in New-Jersey currency?
Answer, £855 17s. $8\frac{1}{7}$d.

7. *Boston, July 26th,* 1837.

Exchange for £320 10s. 6d.

Twelve days after sight, please pay to Peter Finch, or order, three hundred twenty pounds, ten shillings and six pence, value received, and place the same to the account of your

Obt. Servant—
ISAAC WATERFORD, Jr.

To Peter Finch.

Questions. 1. What is Domestic Exchange? 2. Rule of statement?

What sum in New-York currency will discharge this bill? *Answer*, £427 7s. 4d.

8. What sum South-Carolina currency is equal to £429 7s. 3d. in New-England? *Answer*, £333 18s. 11½d.

9. What sum in Pennsylvania, is equal to £259 15s. 9d. Georgia currency? *Answer*, £417 10s. 3d. 2qrs.

10. *Philadelphia, June 1st*, 1837.

EXCHANGE FOR £240 10s. PENNSYLVANIA CURRENCY.

Sixteen days after sight, pay to GEORGE SIMPSON, or order, two hundred and forty pounds, ten shillings, Pennsylvania currency, as per advice from

Yours, &c.

JOSIAH LITTLE.

To THOMAS SMART, *Merchant, N. Y.*

What sum New-York currency will discharge the above bill? *Answer*, £256 10s. 8d.

FOREIGN EXCHANGE.

ALL foreign coins, by a late act of Congress of the United States, are prohibited being a lawful tender. The gold coins of Great Britain and Portugal, of their present standard, are valued at the rate of 100 cents for every 27 grains, or 88⅘ cts. per dwt. The gold coins of France, of their present standard, are to be valued at the rate of 100 cents for 27½ grains, or 87¼ cts. per dwt. The gold coins of Spain, of their present standard, are to be valued at the rate of 100 cents for 28½ grains, or 84 cts. per dwt.

In England, Ireland, and the English West-India islands, accounts are kept in pounds, shillings, pence and farthings; though the intrinsic value, in each place, is not the same.

Exchange is said to be at par between two countries, or states, when the money given in one is equivalent in value to that received for it in another.

The course of exchange is fluctuating, being above or below par, according to the occurrences of trade, or the demand for money.

QUESTIONS. 1. What is foreign exchange? 2. When is exchange said to be at par? 3. What is the meaning of *par*? Answer: It is a *Latin* word, which signifies *equal*.

The following tables show the par value of foreign money in the United States.

TABLE IV.

Value of foreign coins in federal money, as established by a late act of Congress.

	D.	C.	M.
Pound Sterling, - - - - - - - - - -	4	44	4
Pound, of Ireland, - - - - - - - - - -	4	10	0
Pagoda, of India, - - - - - - - - -	1	94	0
Tale, of China, - - - - - - - - - -	1	48	0
Mill-ree, of Portugal, - - - - - - -	1	24	0
Ruble, of Russia, - - - - - - - - - -	0	66	0
Rupee, of Bengal, - - - - - - - -	0	55	5
The Guilder, of the United Netherlands,	0	39	0
Mark Banco, of Hamburgh, - - - - -	0	33	5
Livre Tournois, of France, - - - -	0	18	5
Real Plate, of Spain, - - - - - - - -	0	10	0

TABLE V.

Coins current in the United States, with their Sterling and Federal value.

NAMES OF COIN.	Standard weight	Sterling money of G. Britain.	N. E. States.	N. York and N. Carolina.	N.Jers'y Penn., Del. & Maryl'd	S. Carolina & Georgia.	Federal value.
Gold.	*pwt.g*	*L. s. d.*	*L. s. d.*	*L. s. d.*	*L. s. d.*	*L. s. d.*	*D. C.M.*
A Johannes,	18 [illegible]	3 12 0	4 16 [illegible]	6 8 0	6 0 0	4 0 [illegible]	16 00 0
A half Johannes	9 [illegible]	1 16 0	2 8 [illegible]	3 4 [illegible]	3 0 0	2 0 0	8 00 0
A Doubloon,	16 21	3 6 0	4 8 0	5 16 [illegible]	5 12 6	3 10 0	14 93 3
A Moidore,	6 1[illegible]	1 7 0	1 16 0	2 8 [illegible]	2 5 0	1 8 0	6 00 0
An Eng. Guinea	5 [illegible]	1 1 0	1 8 0	1 17 [illegible]	1 15 0	1 1 9	4 66 7
A French do.	5 [illegible]	1 1 0	1 7 [illegible]	1 16 0	1 14 6	1 1 5	4 60 0
A Span. Pistole,	4 [illegible]	0 16 6	1 2 0	1 9 0	1 8 0	0 18 0	3 77 3
A Fren. Pistole,	4 4	0 16 0	1 2 0	1 8 0	1 7 6	0 17 6	3 66 7
Silver.							
An English or French Crown,	18 0	0 5 0	0 6 8	0 8 9	0 8 3	0 5 [illegible]	1 10 0
The Dollar of Spain, Sweden or Denmark,	17 6	0 4 6	0 6 0	0 8 0	0 7 6	0 4 8	1 00 0
An Eng. Shilling,	3 18	0 1 0	0 1 4	0 1 9	0 1 8	0 1 1	0 22 2
A Pistreen,	3 11	0 0 10¾	0 1 2	0 1 7	0 1 6	0 0 11	0 20 0

All other Gold Coins of equal fineness, at 89 cents per pwt., and Silver, at 111 cents per oz.

TABLE VI.

Monies of different Countries.

FRANCE.

12 deniers=1 sol.
20 sols=1 livre=18½ cts.
3 livres=1 crown.

NOTE. The above is according to the old system; the present method of keeping accounts in France, is in francs and centimes, or hundredth parts, thus:

10 centimes=1 decime.
10 decimes=1 franc=$,1873125.

The 5 franc piece weighs 25 grammes, or 386,1 grains Troy, and is equal in value to $,9365625.

SPAIN.

The money of Spain is of two kinds; one is called vellon, the other plate money. Accounts are most generally kept in rials and marvadies vellon.

4 marvadies vellon, or
2⅛ marvadies of plate, } =1 quarta.
8½ quartas, or
34 marvadies vellon, } =1 rial vellon.
15 rials vellon, - - - - - - =1 peso, or current dollar.
16 quartas, or
34 marvadies of plate, } =1 rial of plate=10 cts.
8 rials of plate, - - - - =1 piastre=80 cts.
10 rials of plate, - - - - =1 dollar=$1,00.
5 piastres, - - - - - - - - =1 Spanish pistole=$4.

ITALY.

12 deniers=1 sol.
20 sols=1 livre.
5 livres=1 piece of eight at Genoa.
6 livres=1 do. " at Leghorn.
6 solidi=1 gross.
24 grosses=1 ducat.

PORTUGAL.

400 reas=1 crusado.
1000 reas=1 millrea=$1,24.

The reas and millreas are imaginary pieces of money. the real moneys of Portugal are as follows:—

Silver.

1 crusado =400 reas=50 cts.
12 vintin piece=280 reas=30 cts.
5 do. =100 reas=12½ cts.
2½ do. = 50 reas= 6¼ cts.

Gold.

1 double johannes=25 millreas, 600 reas=$32.
1 single do. =12 do. 800 reas=$16.
half do. = 6 do. 400 reas=$8.
quarter do. = 3 do. 200 reas=$4.
eighth do. = 1 do. 600 reas=$2.
festoon or $\frac{1}{16}$ do.= 800 reas=$1.
1 moidore = 4 do. 800 reas=$6.

HOLLAND.

8 pennings=1 groat - - - - =01 ct.
2 groats =1 stiver - - - - =2d. or 2 cts.
6 stivers =1 shilling - - - =12 cts.
20 stivers =1 florin or guilder =40 cts.
2½ florins =1 rix dollar - - - =$1,00.
6 florins =1£ Flemish - - =$2,40.
5 guilders =1 ducat - - - - =$2,00.

DENMARK.

16 schillings=1 mark =$0,33⅓.
3 marks =1 rix dollar =$1,00.
6¼ marks =1 ducat =$2,08⅓.

RUSSIA.

3 copecs=1 altima.
10 do. =1 grivena.
50 do. =1 politin.
2 politin=1 ruble=75 cts.
2 rubles=1 ducat.

CHINA.

10 caxa =1 candareen=$,0148.
10 candareens=1 mace =$,148.
10 mace =1 tale =$1,48.

BARBARY.

10 aspers =1 rial =$0,12½.
2 rials =1 double = ,25 cts.
4 doubles=1 dollar =$1,00.
24 medins=1 chequin= ,75 cts.

32 do. =1 dollar =$1,00.
180 aspers =1 zequin =$2,25.
15 doubles=1 pistole =$3,75.

TURKEY.

3 aspers=1 para=$0,00652925.
40 paras=1 piastre=$0,26117647.

BILLS OF EXCHANGE.

To find the value of Bills of Exchange above par.

EXAMPLES.

1. What is the value of a bill of exchange for $860, at 5 per cent. above par?

Operation.

What value $	860 $
If $100	105 $
$	903 *Answer.*

2. A, of Boston, is indebted to C, of London, £1000. How much sterling must be remitted, exchange being 50 per cent.? *Answer*, £1500.

3. B, of New-York, is indebted to D, of Liverpool, £650 sterling, to discharge which, he purchases a bill at 3 per cent. above par. How many dollars does he give for it? *Answer*, $6360,25.

To find the value of Bills of Exchange below par.

1. What sum sterling money is equal to £340 6s. 4d. Massachusetts currency—exchange 40 per cent.?
Answer, £243 1s. 8d.

2. D, in Philadelphia, owes E, in London, £600 sterling, to discharge which he purchases a bill, at 3 per cent. below par. How many dollars must he give?
Answer, $2588,738.

3. *Boston*, ———.

EXCHANGE FOR £540 8s. 9d. STERLING.

At thirty days' sight, pay to TIMOTHY DICKS, or order, five hundred forty pounds, eight shillings and nine pence, value received; and place the same to the account of **JAMES STRIKER.**

To **John Johnson**, Merchant,
Liverpool.

What is the value of this bill in Pennsylvania currency, exchange at 56 per cent.?

4. A, of Cork, draws upon B, of London, for £870 12s. 4d., Irish, exchange at 8 per cent. How much, sterling, will discharge this bill?

5. Exchange for 2446 livres, 6 sols, 4 derniers.

Thirty days after sight of this my second of exchange, first of same tenor and date not paid, pay to Titus True, or order, two thousand, four hundred, forty-six livres, six sols, four deniers, value received, and place the same to my account. **PETER J. TUTTLE.**

To **Trustram Crocker**, Merchant,
Paris.

How much sterling is the above bill, and how much in Pennsylvania currency?

6. A merchant in Toulon is indebted to a merchant in Boston 8462 francs, 20 centimes; what is the amount in federal money?

7. A, of Albany, buys a draft on C, of Paris, of 8846 francs, 34 centimes, for $1600. What is the rate of exchange?

8. Q, of Barcelona, is indebted to P, of New-York, 925 piastres, 3 rials, 24 marvadies plate. How much, in federal money, is Q charged in P's book? *Ans.*, $740,37.

9. C, of Ireland, remits to D, of London, £345 10s. Irish. With how much sterling must C be credited, exchange being 8 per cent.? *Ans.*, £319 18s. 1d. 3$\frac{1}{9}$qr.

10. A bill for 3625 pesos, 4 rials, 31 marvadies, being remitted to Cadiz, what sum New-Jersey money is equal to it, at 7s. 6d. per peso? *Ans.*, £1359 19s. 10d. 3qr.

11. A Virginia merchant shipped tobacco to Norway, worth £1673 18s., Virginia currency. How many rix dollars, at 6s. each, must he receive? *Ans.*, $5579,666.

12. A, in Philadelphia, owes B, of Amsterdam, $750. How many guilders is it, at 40 cts. per guilder?

Answer, 1875.

13. What sum must be paid in Savannah, for an invoice of goods charged at 490 florins, 15 stivers, allowing the exchange at 40 cents per florin, and freight and duties 30 per cent.? *Ans.*, 6255,19.

14. A merchant in Philadelphia receives of a merchant in Amsterdam an invoice of goods, amounting to 12340 florins, 19 stivers, 12 pennings. How much must be remitted, in Pennsylvania currency, to discharge the bill, at 35¼ per florin, and what sum in sterling, exchange at 38s. 6d. Flemish per pound sterling?

Answer, £941 12s. 0d. 0qrs. sterling.

15. In 16745 marks, how many dollars, allowing 33⅓ cts. per mark? *Answer*, $5581,666.

16. In 2045 piastres, 9 rials plate, how many dollars?

Answer, $1636,90.

17. What will 8400 arsheens of ravens duck cost, at 15 rubles for 45 arsheens, in rubles and also in federal money? *Answer*, 2800 rubles : $2100.

18. A, of Bordeaux, draws on B, of Liverpool, for 1400 crowns, at 55d. sterling per crown; for the value of which, B draws again on A, at 56d. sterling per crown; besides commission, of ½ per cent., what did A gain or lose by this transaction? *Answer*, gained 15½ crowns.

19. In $1820, how many pagodas of India?

20. In $605 how many rupees of Bengal?

21. How many dollars in 4678 tales, 8 mace, 7 candareens?

22. In 8000 aspers, how many dollars? *Ans.*, $100.

23. A merchant in Philadelphia imported from England 700 ells of cloth, at 5 shillings sterling per ell. The cost of transportation and duty, on the whole amount, was 35 per cent., the exchange at par. For how many cents must 1 yard be sold in Philadelphia, to gain 12½ per cent?

Answer, 134+ cents.

REDUCTION OF CURRENCIES.

Reduction of Currencies *teaches to reduce pounds, shillings, pence, &c., to federal money, and the reverse.*

RULE.

Reduce the dollar to the fraction of a pound; and, if there be shillings, pence and farthings in the given sum, reduce them to the decimal of a pound, by Inspection, and proceed as in the Rule of Three.

1. In £63, New-England and Virginia currency, how many dollars?

Operation.

How many $	$\cancel{6}\cancel{3}$ £ 21
If £ $\cancel{3}$	10
make	1$
	$210 *Answer.*

If $1 is $\frac{3}{10}$£, then, it is evident that the quotient of £1 divided by $\frac{3}{10}$ would be the number of dollars in one pound, and so of any number of pounds.

3. In £240 10s. Virginia, &c., currency, how many dollars, cents and mills?

5. Reduce £210 15s. Virginia, &c., currency, to federal money.

7. What sum in federal money is equal to £300 10s. 6d., New-England and Virginia currency?

9. Reduce £380 9s., North Carolina currency, to federal money.

11. Reduce £67 5s. 3d., New-Jersey currency, to federal money.

13. What sum in federal money is equal to £67 5s. 3d., South-Carolina and Georgia currency?

15. Reduce £102 15s. 6½d. South-Carolina, &c., currency, to Federal money.

2. In $210, how many pounds, New-England and Virginia currency?

Operation.

How many £	21\cancel{0}$
If $ 1	3£
$\cancel{1}\cancel{0}$	
	£63 *Answer.*

If £$\frac{3}{10}$ is $1, then the product of £$\frac{3}{10}$ multiplied by any number of dollars, will be the number of pounds required.

4. In $801,66,6,+ how many pounds and shillings, New-England, &c. currency?

6. Reduce $702,50 to Virginia, &c. currency.

8. What sum in New-England and Virginia currency, is equal to $1001,75?

10. Reduce $951,12,5 to N. Carolina currency.

12. Reduce $179,36,6,+ to New-Jersey currency.

14. What sum in South-Carolina and Georgia currency is equal to $288,26,5?

16. Reduce $440,85,9 to South-Carolina, &c., currency.

QUESTIONS. 1. What does Reduction of Currencies teach? 2. Rule for reducing pounds, shillings and pence to Federal Money? 3. Rule for reducing Federal Money to pounds, shillings, pence, &c.?

17. Reduce £630 6s. 4½d., Pennsylvania currency, to Federal money.

18. Reduce $1680,85 to Pennsylvania, &c., currency.

19. Reduce £600 10s. 6d. sterling, to Federal money, the dollar being 4s. 6d.

20. Reduce $ 2669, to sterling money.

21. What sum in Federal money is equal to £126 14s. Canada and Nova Scotia currency, the dollar being 5s.

22. What sum in Canada and Nova Scotia is equal to $506,80.

23. Reduce £346 16s., N. York, &c., currency, to Federal money.

24. Reduce $867 to New-York and North-Carolina currency.

25. Reduce £125 7s. 9d., Maryland, &c., currency, to Federal money.

26. Reduce $334,36,8,+ to Maryland, &c., currency.

27. Reduce £501 3s. 9d., Massachusetts currency, to Federal money.

28. Reduce $1670,62,6, to Massachusetts currency.

29. Reduce 450 pence, N. Jersey, &c., currency, to cents. (See Table III.)

Operation.

How many cents. | 450 pence.
d. 9 | 10 cents.
500 cts. *Ans.*

30. Reduce 500 cents, to New-Jersey currency.

Operation.

How many pence. | 500 cents.
Cts. 10 | 9d.
450 pence.

31. Reduce 540 pence, N. England currency, to Federal money.

32. Reduce 750 cents, to pence, New-England currency.

INTEREST.

Interest is a premium paid, or an allowance made, by the borrower to the lender, for the use of a certain sum of money.

The money lent, upon which interest is to be received, is called the *principal.*

Questions. 1. What is Interest? 2. What do you understand by the principal?

The premium paid for the use of the *principal*, is called the *interest.*

The sum paid on $100, or 100 cents, or £100 per annum, is called the *rate* per cent., or per centum. (*Per centum* signifies by the hundred; *per annum,* by the year.)

The principal and interest added together, is called the *amount.*

NOTE. The rate of interest established by law in the New-England States is 6 per cent. In New-York, the legal interest is 7 per cent. In England, it is 5 per cent. When the rate is not mentioned in this Work, 6 per cent. is understood.

Interest is either *simple* or *compound.*

SIMPLE INTEREST.

Simple interest is that which arises from the principal only.

32. What is the interest of $12 for 1 year, at 6 per cent?

Operation.
$12
,06
———
,72*Answer.*

If the interest of $1, for 1 year, be ,06 cents, or $\frac{06}{100}$ of a dollar, then the interest of $12 would be 12 times ,06, or ,06×12=72 cents.

NOTE. The rate per cent. is written as so many hundredths of a dollar: Thus, 6 per cent. is written ,06; 7 per cent., ,07; 5 per cent., ,05; 2 per cent., ,02. It is evident that the rate per cent. must be written so many hundredths, because, being so many cents on every 100 cents, it is so many 100ths of a dollar.

EXAMPLES.

1. What is the interest of $30 for 2 years, at 5 per cent.?

Operation.
$30
,05
——
1,50
2
——
$3,00

If the interest of $1 for 1 year, at ,05 per cent. be $5, then the interest of $30 will be 5×30=1,50, and for 2 years $1,50×2=$3,00. Hence, to compute the interest for 1 or more years, we have the following

RULE.

Multiply the principal by the rate, expressed as the decimal of a dollar, and the product will be

---

QUESTIONS. 3. What do you understand by the rate per cent? 4. What does *per cent.* signify? What, *per annum*? 5. What is the *amount*? 6. What is the legal rate of interest in New-England? 7. What in New-York? 8. What is *simple* interest? 9. Rule to obtain the interest for one year? 10. Why is the rate per cent. written so many hundredths of a dollar?

the interest for 1 year. When the time is more than 1 year, multiply the interest for 1 year by the number of years.

2. What is the interest of $45 for 1 year, at 6 per cent.? *Answer*, $2,70.

3. What is the interest of $22,25 for 1 year, at $5\frac{1}{2}$ per cent.?

Reduced to a decimal.

$05\frac{1}{2}=\frac{11}{2}=,055$

```
   Operation.                 Operation.
   $22,25    Or thus:    ½)$22,25
     ,055                      ,05½
   -------               ----------
    11125                     11125
   11125                       1112
   -------               ----------
 $1,22375   Ans.            $1,2237
```

NOTE. The decimals below mills are not regarded in the answer in this, or the following questions.—For pointing the product, see *Multiplication of Decimals*.

4. What is the interest of $62,75 for 2 years, at 3 per cent.? *Answer*, $3,765.

5. What is the interest of 535,42, for 4 years, at 2 per cent.? *Answer*, $42,833.

6. What is the interest of $115,67,5 for 1 year, at $7\frac{1}{2}$ per cent.? at $6\frac{1}{2}$ per cent.? at $8\frac{3}{4}$? at $9\frac{1}{2}$? at $12\frac{2}{3}$ per cent.?

7. What is the interest of $450,50 for 3 years, at 6 per cent.? *Answer*, $81,09.

In the preceding examples, the interest has been computed for 1 or more years; but it is often necessary to calculate the interest for months and days. Now, as the interest on $1, at ,06 per cent. for 1 year, or 12 months, is 6 cents, it is evident, that it amounts to half a cent a month, or 12 half cents a year on a dollar. If, therefore, we multiply any number of dollars by half the number of months, we shall have the interest for 1 year in cents. Again: as 1 month is 30 days, and the interest for 1 month is $\frac{1}{2}$ cent, or 5 mills, for 1 day it would be $\frac{5}{30}=\frac{1}{6}$ of a mill. If, therefore, we multiply by $\frac{1}{6}$ of the days, we have the interest in mills; or, we may reduce the days to the fraction of a month, and multiply by half the fraction.

8. What is the interest of $40 for 1 year, 6 months, and 5 days?

☞ *For Operation, see next page.*

QUESTIONS. 11. What is the rule for pointing the product?

Operation.

$\frac{5}{6}=\frac{1}{3}$ & $\frac{1}{2}$)$40
,09$\frac{5}{6}$
———
360
20
13$\frac{1}{3}$
———
$3,93$\frac{1}{3}$, *Ans.*

If the interest of $1 for 12 mo. be ,06 cents, the interest for 6 mo. will be ,03 cents, and for 5 days, $\frac{5}{6}$ of a mill; therefore, the interest of $1 for 12 months, 6 months and 5 days, will be ,06+,03+$\frac{5}{6}$=,09$\frac{5}{6}$; the same as one half the months, and one sixth of the days. Hence the

RULE.

When there are months and days in the given time—Multiply by half the number of months in the whole time, and one sixth of the days. If there be an odd month, call it 30 days—to which add the odd days, if any; and dividing them by 6, write the quotient in the place of mills, in the multiplier.

NOTE. If the interest is required for a number of years, multiply the interest for 1 year by the number of years, and compute the interest for the months and days as above directed.

EXAMPLES.

9. What is the interest of $275, for 2 years, 5 months, 6 days? $40,15.

10. What is the interest of $749,605 for 3 years, 7 months, 15 days?

43. What is the interest of $342 for 1 month, 15 days?

Operation.
2)342
,007$\frac{1}{2}$
———
2,394
171
———
$2,565 *Ans.*

NOTE. As there is no even number of months, we supply the two first decimal places with ciphers, as a guide in pointing the product.

11. What is the interest of $678,59, for 1 year, 3 months, 11 days?

12. What is the amount of $678,59, on interest for 1 year, 3 months and 11 days? $730,628.

NOTE. The *amount*, is the principal and interest added together.

13. What is the interest of $600 for 27 days?

14. What is the amount of $750,60, on interest for 18 months and 18 days?

QUESTIONS. 12. What is the rule for computing interest for months and days? 13. Why do we multiply by one half the months, and one sixth of the days?

15. What is the amount of $1000, on interest for 4 years and 6 months? *Ans.*, $1270.

16. A note for $450 on interest, was dated January 1st, 1835. What was due, principal and interest, March 16th, 1837? *Answer*, $509,625.

yr. mo. d.
1837—3—16
1835—1—1

2—2—15 time.

17. A note for $60,50 on interest, was dated Dec. 20, 1834; what was there due, principal and interest, Jan. 28, 1837? *Ans.*, $68,143.

18. What is the amount of $879,30 on interest 2 years, 5 months and 19 days? *Ans.*, $1009,582.

19. What is the interest of $375 for 7 days? *Ans.*, $,437.

20. What is the interest of $89,285 for 1 year, 7 months and 29 days? *Ans.*, $8,913.

21. What is the interest of $336 for 5 months and 16 days? *Answer*, $9,296.

22. What is the amount of $1844,48 on interest 2 months and 21 days? *Answer*, $1869,38.

23. What is the amount of $2731,50 on interest 3 years, 9 months and 26 days? *Answer*, $3357,913.

24. What is the amount of $1764, on interest from June 14, 1829, to July 14, 1837? *Ans.*, $2619,54.

25. What is the interest of £240 8s. 6¾d., for 1 year?

Operation.
£240,428
,06
14,42568
=£14 8s. 6d. *Ans.*]

Reduce the shillings, pence and farthings to the decimal of a pound, by Inspection; (see p.115) then proceed as in Federal Money. The interest will be in pounds and decimal parts, which must be reduced to shillings; (see p. 115.)

26. What is the interest of £379 15s., for 1 year and 6 months? *Ans.*, £34 3s. 6½d.

27. What is the interest of £416 12s. 6d., for 10 months? *Ans.*, £20 16s. 7½d.

28. What is the interest of £427 13s. 9d. 2qrs., for 1 year and 8 months? *Ans.*, £42 15s. 4½d.

QUESTIONS. 14. What is the rule for computing interest on pounds, shillings pence &c.?

29. What is the interest of £129 7s. 3d. 3qrs., for 3 years, 7 months and 5 days? *Ans.*, £27 18s. 4½d.

30. What is the amount of £320 10s. 6d., on interest for 2 years, 6 months and 15 days? *Ans.*, £369 8s. 1½d.

31. What is the amount of £430 7s. 8d. 3qrs., for 4 years, 3 months and 20 days? *Ans.*, £111 3s. 7d. 3qrs.

When the rate of interest is any other than six per cent., and the time consists of years, months and days,

Find the interest first for 6 per cent., and then for 1 per cent., and multiply the interest at 1 per cent. by the given rate, and the product will be the answer.

EXAMPLES.

32. What is the interest of $680 for 1 year and 6 months, at 7 per cent.? *Answer*, $71,40.

Operation.

$680
,09

6)61,20 interest at 6 per cent.

10,20 interest at 1 per cent.
7

$71,40 interest at 7 per cent.

33. What is the interest of $336,40 for 2 years, 8 months and 3 days, at 3 per cent.? *Ans.*, $26,996.

34. What is the interest of $556,36 for 3 years, at 1 per cent.? *Answer*, $16,69.

NOTE. The interest of any sum at 1 per cent. for 1 year, is the principal itself, with the separatrix moved two figures further towards the left; therefore, to obtain the interest at 1 per cent. for any number of years, we have only to multiply by the number of years.

35. What is the interest of $0,56 cents for 5 years, 5 months and 10 days, at 9 per cent.? *Ans.*, $,276.

36. What is the amount of $1000, on interest for 5 years 7 months, at 7½ per cent? *Answer*, $418,16.

37. What is the interest of $1569,20 for 1 year, at 1 per cent.? *Answer*, $15,692.

QUESTIONS. 15. Rule, when the rate of interest is any other than 6 per cent.? 16. What is the interest of any sum for 1 year at 1 per cent.?

INTEREST BY CANCELLING.

RULE.

State the question as in Direct Proportion, by placing the terms of demand on the right, and the terms of supposition on the left.

EXAMPLES.

1. What is the interest of $500 for 3 years, at 6 per cent.?

Note I. The terms of supposition in Interest, are not expressed, being always 100 and 1 year. The foregoing question may be expressed thus:

What is the interest of $500 for 3 years, if the interest of $100 for 1 year be $6?

Operation.

$$\begin{array}{r|l} \text{What interest.} & 5\cancel{0}\cancel{0}\ \$ \\ \text{If } \$1\cancel{0}\cancel{0} & 3\text{ years.} \\ \text{Year } 1 & 6\ \$ \\ \hline \$ & 90\ \textit{Answer.} \end{array}$$

2 What is the interest of $720 for 1 year and 6 months, at 6 per cent.? *Answer*, $64,80.

Note II. When the given time is months, weeks, or days, either less or greater than a year, reduce it to the lowest denomination, and 1 year, the time in the supposition, to the same denomination.

3. What is the interest of $642,255 for 2 years and 6 months? *Answer*, $96,338.

4. What is the interest of $1000,68 for 2 months and 15 days? *Answer*, $12,508.

5. What is the interest of $440 for 4 years, at 4 per cent.? *Answer*, $70,40.

6. What is the interest of $60,10 for 5 years, at 5 per cent? *Answer*, $15,025.

7. What is the interest of $160 for 36 days, at 7 per cent.? *Answer*, $1,12.

8. What is the amount of $780 for 3 years and 4 months, at 3 per cent.? *Answer*, $858.

Note III. If we multiply the amount of $1 for the given time, by the given principal, the result will be the same as adding the principal to the interest. Thus—the amount of $1 for 3 years and 4 months, at 3 per cent., is $1,10, which multiplied by $780, gives $858, the answer.

I. *When time, rate and amount are given, to find the principal.*

1. What principal will amount to $858 in 3 years and 4 months, at 3 per cent. ?

The student will perceive, that this question is the reverse of question 8th, preceding, and also, that 858 is there a product, of which 1,10, the amount of $1 for the given time, is a factor; therefore, if we divide 858 by 1,10, we shall obtain the other factor, or the principal required—858÷1,10=$780, the answer. Hence the

RULE.

Divide the given amount by the amount of $1 for the given time, and the quotient will be the answer.

EXAMPLES.

2. What principal will amount to $778,10, in 4 years and 3 months, at 6 per cent. ? *Answer*, $620.

3. What principal will amount to $650, in 6 years, at 5 per cent. ?

Operation by Cancelling.

What principal.	650$ amount.
Amount, $1,30	1$ principal.
	$500 *Answer.*

4. What principal will amount to $738,40, in 7 years? *Answer*, $520.

II. *When time, rate and interest are given, to find the principal.*

1. What principal will gain $27,52, in 1 year, 5 months and 6 days?

We have seen, that the interest of a given principal for a given time, is the product of the interest of $1 for the same length of time, and the principal; therefore, if we divide $27,52 by ,08,6, the interest of $1 for the given time, we shall obtain the principal required, as before.—Hence the Rule:

Divide the given interest by the interest of $1 for the given time, and the quotient will be the answer.

2. What principal will gain $19 in 4 months, at 6 per cent. ? *Answer*, $950.

3. What principal will gain $1500 in 5 years, at 6 per cent.? *Answer*, $5000.

III. *When principal, interest and time are given, to find the rate per cent.*

1. If $50, in 6 months, gain $1,50, what is the rate per cent.?

If the interest of $50, at 1 per cent., be 25 cents, then the quotient of $1,50, the whole interest, divided by 25 cents, will be the rate per cent. required. 1,50÷25=6 per cent., answer. Hence the RULE:

DIVIDE the given interest by the interest on the given principal, at 1 per cent. for the given time, and the quotient will be the answer.

2. If $300 gain $12 in 8 months, what is the rate per cent.?

Operation by Cancelling.

What interest.	100 $
If $300	12 m.
m. 8	12$ int.

Answer, 6 per cent.

3. If $740 gain $27,75 in 9 months, what is the rate per cent.? *Answer*, 5 per cent.

4. If $1000 gain $75 in 6 months, what is the rate per cent.? *Answer*, 15 per cent.

IV. *When principal, rate and interest are given, to find the time.*

1. In what time will $300 gain $12, at 6 per cent.

Operation.

In what time.	12$ Int.
Int. $18	1 year.
3	2=8 months.

Having found the interest of $300 for 1 year, the question may be expressed thus: In what time will $12 interest be gained, if $18 be gained in 1 year? It is evident, that the ratio of the interest for 1 year is to the given interest as 1 year is to the time required. Hence the RULE:

DIVIDE the given interest by the interest of the given principal for 1 year, and the quotient will be the answer.

2. In what time will $240 gain $4,80, at 6 per cent? *Answer*, 4 months.

3. In what time will $600 amount to $645, at 5 per cent.? *Answer*, 1 year and 6 months.

4. In what time will $375 gain $28,12½, at 6 per cent.?
Answer, 1 year and 3 months.

5. The interest on a note of $225, at 4 per cent., was $11,40. What was the time?
Answer, 1 year, 3 months, 6 days.

To find the interest on notes, &c., when partial payments have been made.

The methods of computing interest on notes and bonds differ in different places; but the following is the more general method, and is established by law in Massachusetts.

Compute the interest up to the time of the first payment; and if the payment exceed the interest, deduct the excess from the principal, and cast the interest on the remainder up to the second payment, and so on. If the payment be less than the interest, cast the interest up to the time when the sum of the payments shall exceed the interest; then deduct the excess from the principal, and proceed as before.

$3784,25.

1. For value received, I promise to pay James Larned, or order, three thousand, seven hundred, eighty-four dollars and twenty-five cents, with interest.

JOHN FISHER.

July 10, 1826.

On this note were the following endorsements:

Jan. 16, 1827,	received	$148,21
Aug. 11, 1827,	"	50,00
Dec. 24, 1828,	"	2789,25
Feb. 12, 1830,	"	1000,00

What was due Dec. 14, 1830? *Answer*, $464,867.

☞ *For Operation, see next page.*

Questions. 17. Method of computing interest on notes, &c., established by law in Massachusetts?

The first principal, on interest from July 10, 1826,		$3784,25
Interest to Jan. 16, 1827, time of the first payment, (6 months, 6 days,)		117,311
		$3901,561
Payment exceeding the interest, Jan. 16,		148,21
Remainder for a new principal,		$3753,351
Interest from Jan. 16, 1827, to Dec. 24, 1828, (1 year, 11 months, 8 days,)		436,639
Payment, Aug. 11, less than the int.	$ 50,00	
Payment Dec. 24, exceeds the int.	2789,25	
Sum of the payments,	2839,25	4189,990
		2839,250
Remainder for a new principal,		$1350,740
Interest from Dec. 24, 1828, to Feb. 12, 1830, (1 year, 1 month, 18 days,)		91,850
		1442,590
Payment, Feb. 12, exceeds the interest,		1000,000
Remainder for a new principal,		$442,590
Interest from Feb. 12, 1830, to Dec. 14, 1830, (10 months, 2 days,)		22,277
Balance due Dec. 14, 1830,		$464,867

$6420,50

2. For value received, I promise to pay THOMAS TERRIL, or order, six thousand, four hundred, twenty dollars, and fifty cents, with interest.

May 4, 1830. SAMUEL ENGLISH.

On this note were the following endorsements :

March 4, 1831,	received	$ 40,00
Dec. 1, 1831,	"	200,00
Feb. 10, 1832,	"	5000,00
June 28, 1833,	"	1534,25

What was the sum due March 1, 1834?

Answer, $500,784

3. A's note of \$374,62 was given Jan. 1, 1834, on interest after 90 days. June 4, 1836, he paid \$320. What was due August 15, 1837? *Answer*, \$110,90,7.

4. B's note of \$654,32, was given Dec. 12, 1831, on which was endorsed the interest for 18 months and 4 days. What was due on settlement, Nov. 20, 1833?

Answer, \$671,11,4.

COMMISSION, BROKERAGE & INSURANCE.

COMMISSION and BROKERAGE are compensations of so much per cent. to factors and brokers, for their respective services in buying and selling goods, &c.

Insurance, is an exemption from hazard, obtained by the payment of a certain sum, which is generally so much per cent. on the estimated value of the property insured.

Premium, is the sum paid by the owner of the property, for the insurance.

Policy, is the name given to the instrument, or writing, by which the contract of indemnity is effected between the insurer and insured.

The Policy should always cover a sum equal to the estimated value of the property insured, together with the premium; that is, a policy to secure the payment of \$100 at 3 per cent., must be made out for \$103.

RULE.

Method of Operation the same as in Simple Interest.

EXAMPLES.

1. If a factor purchase goods to the amount of \$1800, and I allow him $\frac{3}{4}$ per cent. for his services, what must I pay him?

Operation.

$$\begin{array}{r|l} 1\cancel{0}\cancel{0} & \cancel{1}\cancel{8}0\cancel{0}\ \ 9 \\ 2\ \ \cancel{4} & 3 \\ \hline 2 & 27 \\ \hline & \$\ 13\frac{1}{2}\ \textit{Answer.} \end{array}$$

QUESTIONS. 1. What are Commission and Brokerage? 2. What is Insurance? 3. What is a Premium? 4. What is a Policy? 5. What sum should the Policy cover? 6. Give the example.

2. What commission must a factor receive for selling goods to the amount of $864,78, at 4½ per cent.?
Answer, $38,915.

3. What is the commission on $3784,22, at 12½ per cent.? *Answer*, $473,027.

4. A factor buys goods to the amount of $1200; what will be his commission at 1½ per cent.? *Ans.*, $18.

5. What is the brokerage on $9798,67½, at 5¾ per cent.?
Answer, $563,423.

6. The value of a certain ship and cargo is $50000. What is the insurance, at 15 per cent.? *Ans.*, $7500.

7. What is the duty on 4 boxes of tea, each weighing 1 cwt. 2 qrs. 14 lbs., at 1½ cent. per lb.? *Ans.*, $10,92.

8. What may a broker demand on $1000 at 3 per cent.?
Answer, $30.

9. What will be the premium for insuring a ship and cargo, valued at $57840, at 3½ per cent.?
Answer, $2024,40.

10. What may a broker demand on £320 10s. 6d., at 4s. 3d. per cent.? *Ans.*, £67 17s.

11. What will be the premium for insurance on property to the amount of $9248,28, at ½ per cent.? at ¾ per cent.? at ⅓ per cent.? at ⅔ per cent.? at ⅞ per cent.? at ⅘ per cent.?

COMPOUND INTEREST.

Compound Interest is interest upon interest, or that which arises from making the interest a part of the principal, whenever it becomes due.

RULE.

Find the amount of the given principal for the first year, or the first stated time for the interest to become due, by simple interest, and make the amount the principal for the next year, or stated period: and so on to the last. From the last amount, subtract the given principal, and the remainder will be the compound interest required.

EXAMPLES.

1. What is the compound interest of $200 for 3 years, at 6 per cent.?

☞ *For Operation, see next page.*

Questions. 1. What is Compound Interest? 2. What is the Rule?

Operation.

$200 first principal.
,06

12,00 interest. } to be added.
200, principal. }

212, amount, or principal, for 2d year.
,06

12,72 compound interest, 2d yr. } to be
212 principal, " } added

224,72 amount, or principal, for 3d year.
,06

13,48,32 compound interest, 3d year } to be
224,72 principal, " } added

238,20,32 amount.
200 first principal, subtracted.

$38,20,32 the compound interest required.

2. What is the compound interest on a note of $325, on interest 5 years? *Answer*, $109,92.

3. What is the compound interest of $680, for 4 years? *Answer*, $178,479.

◆

TABLE,

Showing the amount of $1, or £1, for any number of years, not exceeding 30 years, at the rates of 5 and 6 per cent. compound interest.

Years.	5 per cent.	6 per cent.	Years.	5 per cent.	6 per cent.
1	1,05	1,06	16	2,18287+	2,54035+
2	1,1025	1,1236	17	2,29201 ‘	2,69277 ‘
3	1,15762+	1,19101+	18	2,40661 ‘	2,85433 ‘
4	1,21550 ‘	1,26247 ‘	19	2,52695 ‘	3,02559 ‘
5	1,27628 ‘	1,33822 ‘	20	2,65329 ‘	3,20713 ‘
6	1,34009 ‘	1,41851 ‘	21	2,78596 ‘	3,39956 ‘
7	1,40710 ‘	1,50363 ‘	22	2,92526 ‘	3,60353 ‘
8	1,47745 ‘	1,59384 ‘	23	3,07152 ‘	3,81974 ‘
9	1,55132 ‘	1,68947 ‘	24	3,22509 ‘	4,04893 ‘
10	1,62889 ‘	1,79084 ‘	25	3,36355 ‘	4,29187 ‘
11	1,71033 ‘	1,89829 ‘	26	3,55567 ‘	4,54938 ‘
12	1,79585 ‘	2,01219 ‘	27	3,73345 ‘	4,82234 ‘
13	1,88564 ‘	2,13292 ‘	28	3,92012 ‘	5,11168 ‘
14	1,97993 ‘	2,26090 ‘	29	4,11613 ‘	5,41838 ‘
15	2,07892 ‘	2,39655 ‘	30	4,32194 ‘	5,74349 ‘

NOTE. Although the decimals, in the preceding numbers, are carried to five places, yet four decimals are generally sufficient for most business operations.

4. What is the compound interest of $650 for 6 years, at 6 per cent.? *Answer*, $272,031.

By the foregoing table we find the amount of $1 for 6 years to be $1,41851; which, multiplied by $650, gives $922,031, the amount of $650 for 6 years, and $922,031 —650=$272,031, the interest required.

5. What is the compound interest of $350 for 2 years and 6 months? *Answer*, $55,057.

NOTE. When there are months and days, first find the amount for the *years*, and on this amount cast the interest for the months and days; this, added to the amount, will give the answer.

6. What is the compound interest of $135 for 3 years, 6 months and 6 days? *Answer*, $30,77.

7. What is the compound interest of $678,25, for 12 years and 6 months, at 5 per cent.? *Ans.*, $570,236.

8. What is the compound interest of $579,75, for 20 years?—for 30 years?

9. What is the amount of a note of $150 for 4 years, at 6 per cent., compound interest? *Answer*, $189,37.

10. The amount of a certain note, at compound interest for 4 years, was $189,37+. What was the principal?

This question, it will be perceived, is the reverse of the last. If the amount required is obtained by multiplying the amount of $1 for the given time by the given principal, then it follows, that if we divide the given amount by the amount of $1 for the given time, we shall obtain the required principal.

11. What is the amount of $597,75, for 20 years, at 6 per cent. compound interest? *Answer*, $1917,077.

12. What is the amount of $1350 for 3 years, at 5 per cent. compound interest? *Answer*, $1562,79,3.

13. What is the amount of a note of $150 for 2 years, compound interest, the interest becoming due at the end of every 3 months? *Answer*, $168,967.

14. What is the compound interest of £240 10s. 6d. for 2 years, at 6 per cent.? *Ans.*, £29 14s. 6d. 3qrs.

15. What is the amount of £450 for 3 years, at 5 per cent. compound interest? *Answer*, £520 18s. 7d.

16. What is the amount of £256 10s. in 7 years, at 6 per cent. compound interest? *Answer*, £385 13s. 7½d.

QUESTIONS. 3. What is the rule, when there are months and days?

DISCOUNT.

Discount is an allowance made for the payment of money before it becomes due.

The *present worth* of a debt due at any future period is so much money as, being put on interest, at a given rate per cent., will amount to the debt, when it becomes due.

1. A holds B's note for $106, due in 1 year, what is the present worth of the note, discounting at 6 per cent.?

It is evident, that if B pays A $106 now, at the end of the year, when the note becomes due, A will have the interest of $106 more than is his due; therefore, B ought to pay him such a sum, as, being put on interest, would amount to $106 at the end of the year. If we divide 106 by the amount of $1 for 1 year, we shall have the principal, or that sum which being put on interest at the usual rate per cent., will amount to the debt when it becomes due. (See p. 160.) $106÷$106=$100, the present worth of $106 due a year hence. From the above we derive the following **Rule**:

To find the Present Worth.

Divide the given sum by the amount of $1 for the given time, and the quotient will be the *present worth.*

The *present worth* subtracted from the debt, will leave the *discount.*

2. What is the present worth of $246,21, payable in 2 years and 8 months, discounting at 6 per cent.? *Answer*, $212,25.

3. How much ready money will purchase a note of $1719,04, due 6 years hence, discounting at 6 per cent.? *Answer*, $1264.

4. Suppose I owe a note of $416 to be paid in 4 years and 2 months, and wish to pay it now, what must be discounted for present payment? *Ans.*, $83,20.

5. How much ready money will purchase a note of $37,165 due 5 years, 1 month and 18 days hence, discounting at 6 per cent.? *Answer*, $28,413+.

6. What is the present worth of a note of $840, payable one half in 10 months, the other half in 20 months, discount 6 per cent. per annum? *Answer*, $781,818.

Questions. 1. What is *discount?* 2. What is the *present worth?* 3. Rule for finding the *present worth?*—for finding the *discount?* 4. What is the difference between *interest* and *discount?*

7. What is the present worth of $1500, due 40 years hence, discount 12 per cent. per annum?

Answer, $258,62+.

8. What is the discount of $420 due in 1 year and 6 months, at 6 per cent.? *Answer*, $34,679+.

9. What is the discount of $109,86 for 1 year, at 6 per cent.? *Answer*, $6,219.

10. Bought goods to the amount of $1909,34, at four months credit; how much ready money must I pay, discount at 3½ per cent.? *Answer*, $1887,321.

11. What is the present worth of £4000, payable in 9 months, at 4¾ per cent. discount?

Answer, £3862 8s. 6d.

The foregoing is the *correct* method of reckoning discount; yet the usual method in practice, is to compute the interest for the time, and deduct it from the given sum. The *interest* thus found is called the *discount*.

The difference between interest and discount on a small sum, for a short time, is inconsiderable, but the difference becomes very considerable when the sum is large, and the time long, for which the discount is to be made.

12. What is the difference between the interest and discount of $100 for 1 month, at 6 per cent.?

Answer, 2½ mills, nearly.

13. What is the difference between the interest and discount of $649 for 3 years at 6 per cent.?

Answer, $17,82.

Bank Discount is the same as *simple interest*. When a note is discounted at a bank, the interest is computed on the sum from the date of the note to the time when it becomes due, including three days of *grace*, and deducted as *discount*. Thus, if a note of $100 be discounted for 30 days, the interest is computed for 33 days. Custom has allowed to the borrower, 3 days after the day on which the note becomes due, called *days of grace;* and as payment is generally withheld until the third day, it is justice that interest should be paid for these days.

If the payment of a note cannot conveniently be made at the proper time, the note may be taken up, if the bank allow the indulgence, by a new note, which must be pre-

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QUESTIONS. 5. What is the usual method in practise? 6. What is *bank discount*? 7. What is meant by *days of grace*?

sented on the day of discount immediately preceding the day on which the note would have become due, paying at the same time the discount, or interest, as before stated. Thus the borrower loses the discount on his note from the day, on which he replaces it by another, to the day on which it would have been to be paid.

The discount of any sum discounted for 30, 60, or 90 days, is found by multiplying by ⅙ of the days. (See p. 156.)

EXAMPLES.

14. What is the bank discount on a note of $714 for 30 days, at 6 per cent.?

Operation.
2)714
,005½
3570
357
$3,927, *Answer.*

15. What is the bank discount on a note of $1692, for 60 days, at 6 per cent.?

Operation.
2)1692
,010½.
16920
846
$17,766, *Answer.*

16. What is the bank discount on a note of $784, for 90 days, at 6 per cent.?

Operation.
2)784
,015½
3920
784
392
$12,152, *Answer.*

17. What is the bank discount on a note of $53, for 30 days? *Ans.*, $,291.

18. What is the bank discount on a note of $1092, for 30 days? *Ans.*, $6,006.

19. What is the bank discount on a note of $2049, for 30 days? *Ans.*, $11,269.

20. A's note of $561, for 60 days, is discounted at the bank, at 6 per cent.? What ready money does he receive? *Ans.*, $555,109.

QUESTIONS. 8. How is the discount found for 30, 60 and 90 days?

21. B's draft for $150, drawn at 15 days' sight, is cashed at the bank, at 3 per cent. discount. How much money does he receive? *Ans.*, $149,813.

22. What is the bank discount on a note of $340, for 90 days, at 6 per cent? *Ans.*, $5,27.

23. What is the bank discount on a note of $632,75, for 90 days, at 6 per cent.? *Ans.*, $9,807.

When a note is offered at the bank for discount, one or two endorsers are generally required; and the note is presented in one of the following forms:

$500. CONCORD, JULY 4th, 1837.

For value received, we, the subscribers, jointly and severally promise to pay the President, Directors and Company of the New-England Bank, or order, five hundred dollars, at said bank, on demand, with interest after sixty days.

When a note called *business paper*, is offered for discount, it is generally made in the following form:

$350. BOSTON, AUGUST 6th, 1837.

Three months after date, I promise to pay to the order of Mr. JOHN SAVAGE, at the Commonwealth Bank, three hundred and fifty dollars, value received.

A. B.

In order to negotiate this note to an individual, or to procure a discount of it at a bank, the said Savage should endorse his name upon the back of the note, and such other names of endorsers should be procured as may be required; in which case, the promisor, or payor, A. B., is first liable for the note, and the note should be demanded of him when it becomes due. If not paid, immediate notice should be given to the endorsers of the note, and on such demand and notice the endorsers become liable for payment of the note; otherwise they are not holden.

The promisor, or payor of a note, is the individual who signs it. The promisee, or payee, is the person to whom the note is payable.

QUESTIONS. 1. When a note is offered at a bank for discount, what is required? 2. What is the form of a note payable to the president, directors, &c. of a bank? 3. What is the form of a note called *business paper*? 4. Who is promissor of a note—the promisee?

When a note is endorsed, the promisee, or payee, is always an endorser.

LOSS AND GAIN.

LOSS AND GAIN teach to find what is gained or lost in the purchase and sale of goods, and also to regulate the price, so as to gain or lose, at a certain rate per cent.

1. If I purchase goods to the amount of \$50, and sell the same for \$60, what do I gain per cent.?

It is evident, that the gain on \$1 would be $\frac{1}{50}$ as much as on \$50. Since, then, the gain on \$50 is \$10, or the gain is $\frac{10}{50}$ of the cost, then \$10÷50=20 cts. on a dollar, or 20 per cent., the answer. Hence the RULE:

When the prices at which goods are bought and sold are given, to find the gain or loss per cent.,

Divide the *gain* or *loss*, found by subtraction, by the *cost* of the article.

EXAMPLES.

2. A merchant bought goods to the amount of \$500, and sold the same for \$700. What did he gain per cent.?

The question may be thus expressed, as in the Rule of Three: What is the gain on \$100, if on \$500 the gain be \$200? *Operation.*

$$\begin{array}{r|l} \text{What gain.} & \cancel{1}\cancel{0}\cancel{0}\ \$ \\ \text{If } \$\ \cancel{5}\cancel{0}\cancel{0} & \cancel{2}\cancel{0}\cancel{0}\ 40 \\ \hline \textit{Answer,} & 40 \text{ per cent.} \end{array}$$

3. A merchant purchased goods to the amount of \$342, 25 cts., and gains on the sale \$41,07; what is the gain per cent.? *Answer,* 12 per cent.

4. Bought flour to the amount of \$840—sold the same for \$907,20; what do I gain per cent.?

Answer, 8 per cent.

QUESTIONS. 1. What is *loss* and *gain*? 2. How is the gain or loss per cent. found? 3. Having the gain or loss per cent., how is the price at which an article is bought or sold found?

5. Suppose a merchant purchase goods to the amount of \$1000, and sell them for \$910, what is the loss per cent.? *Answer*, 9 per cent.

6. Bought fur caps for \$7 apiece; sold them for \$7,25. What was the whole gain in laying out \$630, and what was the gain per cent.?

Answer, $\left\{\begin{array}{l}\text{Whole gain \$22,50.}\\ \text{Gain per cent. 3,57+.}\end{array}\right.$

7. What is the whole loss, and what is the loss per cent., in laying out \$70 for hats, at \$1,75 each, and selling them for 25 cts. apiece, less than cost?

Answer, $\left\{\begin{array}{l}\text{Whole loss \$10.}\\ \text{Loss per cent. } 14\frac{2}{7}\end{array}\right.$

8. Bought 100 yds. of cloth, at \$6,72 per yd., and sold the same for \$8,40. What did I gain per cent.?

Answer, 25 per cent.

When the gain or loss per cent. is given, to find the price at which goods are bought or sold.

RULE.

If the per cent. be *gain*, add it to 100; if the per cent. be *loss*, subtract it from 100, and proceed as in the Rule of Three.

EXAMPLES.

9. A merchant sold cloth, which cost \$6,72 per yard, at 25 per cent. profit. For how much did he sell the cloth per yard? (See Interest, note III., p. 159.)

Operation.

$$\begin{array}{r|l} \text{How many } \$ & \cancel{6}\cancel{7}\cancel{2}\ \$\ 168 \\ \cancel{4}\ \$\ \cancel{1}\cancel{0}\cancel{0} & \cancel{1}\cancel{2}\cancel{5}\ \$\ 5 \\ \hline \$ & 840\ \textit{Answer.} \end{array}$$

10. A merchant sold cloth at \$8,40 per yard, and gained 25 per cent. What was the first cost? (See Interest, p. 160.)

Operation.

$$\begin{array}{r|l} \text{How many } \$ & 840\ \$ \\ \cancel{5}\ \$\ \cancel{1}\cancel{2}\cancel{5} & \cancel{1}\cancel{0}\cancel{0}\ \$\ 4 \\ \hline \$ & 6,72,\ \textit{Answer.} \end{array}$$

11. If 1 tun of wine cost £40, for how much must it be sold, to gain $6\frac{1}{4}$ per cent.? *Answer*, £42 10s.

12. Sold 10 yds. of cloth for £4 16s., and gained 10 per cent. What was the prime cost per yard?

Ans., 8s. $8\frac{8}{11}$d.

13. Bought 7 tuns of wine, at $61,20 per hhd.; sold at 18 cents a pint. What was the whole gain, and how much per cent.? *Answer*, { Whole gain, $826,56. Gain per cent., $48,235.

14. Purchased 40 gallons of molasses, at 3s. per gallon. By accident, 6 gallons leaked out. At what rate must I sell the remainder per gallon to gain 10 per cent. upon the first cost, and give 8 months credit? *Ans.*, 4s. 0d. 1qr.

15. If I sell a pound of silk for $12,72, and gain $1,20, how much should I gain in selling a bale which cost $1152? *Answer*, $108,67$\frac{49}{53}$.

16. Bought 300 lbs. of coffee, at 4s. 2d. per lb., ready money, and sold the same for 5s. per lb., payable in 8 months. How much was gained upon the whole, and how much per cent.? *Answer*, { £12 0s. 4d. $2\frac{6}{13}$qrs. $19\frac{3}{13}$ per cent.

17. Bought 50 yards of broadcloth, at $5 per yard, which I purpose to sell at 25 per cent. profit, ready money; but if I sell it on credit, I must have 5 per cent. extra. How must I sell it per yard, at 6 months, to make both these gains? *Answer*, $6,669.

18. If by selling tea at 57 cts. per lb. I lose 3 cts., what is the loss per cent.? *Answer*, 5 per cent.

19. A merchant purchases 180 casks of raisins, at 16s. per cask, and sells the same at 28s. per cwt., and gains 25 per cent. What is the weight of each cask?

Answer, 80 lbs.

20. What will be the gain in selling $500 worth of flour, at 8 per cent. advance? *Answer*, $40.

21. Bought 1000 bushels of corn for $1922,25. For how much must it be sold to gain 15 per cent.?

Answer, $2210,587.

22. Bought 80 reams of paper, at $2,50 per ream. For how much must the whole be sold to lose 5 per cent.?

Answer, $190.

23. A merchant bought 500 yards of broadcloth for $2125. For how much must he sell the whole, to lose 10 per cent.? *Answer*, $1912,50.

24. If I buy 45 bushels of salt, at 95 cents per bushel, for how much must it be sold per bushel, to gain 20 per cent.? *Answer*, $1,14.

25. Bought 64 bushels of wheat, at $1,75 per bushel. For how much per bushel must I sell it, to lose 3 per cent? *Answer*, $1,697.

STOCK.

Stock is a general name for capital employed in trade, manufactures, insurance, banking, &c. Also, for money loaned to government, or property in a public debt.

The **Capital Stock** of any company, or corporation, is the whole amount originally invested by such company or corporation, which sum is divided into shares, and each holder receives a certificate of the number of shares to which he is entitled. If stock which cost $100 per share sells in the market for any thing more than that amount, it is said to be above *par;* that is, above a sum equal to the first cost—the term *par* signifying equality. If it sells for less than that amount, it is below par; and the amount above or below par, is spoken of as so much per cent. If it sells for $6 on the $100 in advance, it is 6 per cent. above par. If it sells for so much less, it is so much below par.

EXAMPLES.

1. What is the value of $600 of stock, at 106 per cent. above par? *Answer*, $636.

2. What is the value of $2000 of rail-road stock at 87½ per cent.? *Ans.*, $1750.

3. What is the value of $1500 of bank stock at 108 per cent.? at 107 per cent.? at 115 per cent.? at 105 per cent.?

BARTER.

Barter is the exchanging of one commodity for another, according to prices or values agreed upon by the parties.

Questions. 1. What is stock? 2. What is the capital stock of a company, or corporation? 3. When is stock said to be above par? 4. When below par? 5. What is *barter*? 6. Rule?

RULE.

Divide the value of that article whose quantity is given, by the price of the article whose quantity is required; or, the question may be solved by the *Rule of Three.*

EXAMPLES.

1. How many pounds of coffee, at $13\frac{1}{3}$ cts. per pound, must be given in barter for 1200 pounds of sugar, at 8 cents per pound?

Operation.

$13\frac{1}{3}=\frac{40}{3} : 8 :: 1200$
1200
$4|0)960|0$
240
3
720 lbs. *Ans.*

By cancelling.

How many lbs. coff.	$120\cancel{0}$ lbs. sug.
cts. $\cancel{4}\cancel{0}$	$\cancel{8}$ 2
$13\frac{1}{3}=\frac{40}{3}$	3
	1 lb. coffee.
	720 lbs. *Ans.*

2. How much tea, at 64 cents per pound, must be given in barter for 2 cwt. chocolate, at 32 cents per pound?
Answer, 112 pounds.

3. How many pounds of lead, at 9 cents per pound, must be given for 783 lbs. of iron, at 6 cents per lb.?
Answer, 522 lbs.

4. A has broadcloth, at 16s. 6d. per yard; B has linen, at 1s. 4d. per yard. How many yards of broadcloth must be given in exchange for 660 yards of linen?
Answer, $53\frac{1}{3}$ yds.

5. A bartered $53\frac{1}{3}$ yds. of broadcloth, at 16s. 6d. per yard, for 660 yards of linen. What was the price of the linen? *Answer*, 1s. 4d.

6. How much sugar, at 8 cents per pound, must be given, in barter, for $1\frac{3}{4}$ cwt. of cinnamon, at $54\frac{6}{7}$ cents per pound? *Answer*, 12 cwt.

7. A barters $1\frac{3}{4}$ cwt. of cinnamon, at $54\frac{6}{7}$ cents per lb., for 12 cwt. of sugar. What was the value of the sugar per pound? *Ans.*, 8 cents.

8. A has linen worth 20d. per ell English, ready money, but in barter he will have 2s. B has broadcloth worth 14s. 6d. per yard, ready money. What ought to be the price of the broadcloth in barter? *Answer*, 17s. $4\frac{4}{5}$d.

9. B has coffee which he barters with C at 10d. per lb. more than it cost him, for tea which cost 10s.; but in barter C puts it at 12s. 6d. What was the first cost of the coffee? *Ans.*, 3s. 4d.

10. A has 5 tons of butter, $425 per ton, and 10½ tons of tallow, £33 15s. per ton, which he barters with B for 316 barrels of beef, at 21s. per barrel, and the remainder in cash. How much money does he receive?

Ans., $500,25.

11. C and D barter. C has corn, at 75 cents, ready money, but in barter he will have $1. B has rye at 50 cents, ready money. What ought he to have for his rye, in barter? *Ans.*, 66⅔ cents.

12. A has rye at $1,44 per bushel, ready money, but in barter he will have $1,56 per bushel. D has cotton, at 18 cents per pound, ready money. What price must the cotton be in barter, and how many pounds of cotton must be bartered for 100 bushels of rye?

Ans., { Cotton, 19½ cents per pound.
800 lbs. for 100 bushels rye.

13. B gives C 250 yards drugget, at 18½d. per yard, for 308⅓ lbs. of pepper. What does the pepper cost C per lb.?

Ans., 15d.

14. A and B barter. A has 41 cwt. of hops, at $7,20 per cwt., for which B gives him $96 in money, and the rest in prunes, at 10 cts. per lb. What quantity of prunes does A receive? *Ans.*, 17cwt. 3qrs. 4lbs.

15. How many acres of land, worth £40 10s. per acre, must be given for 600 acres worth $8,50 per acre.

Ans., $37\frac{7}{9}$.

16. A has 7½ cwt. of sugar, at 8d. per pound, for which B gives him 12½ cwt. of flour. How much per pound was the flour? *Ans.*, 4⅘d.

17. A has corn, at $1,25, ready money, but in barter he values it at $1,50 per bushel. B has cotton at 20 cts. per pound, ready money. What should be the price of the cotton in barter, and how many pounds must be given for 100 bushels of corn? *Answer to the last*, 625 lbs.

18. A has cloth valued at $4 per yard, ready money, but in barter he will have $4,50. B has cloth at £2 sterling per yard, ready money. At what price ought B to rate his cloth in barter, and how many yards must be given A in exchange for 540 yards of cloth?

Ans. to the last, 324 yards.

19. D has ribbon at 2s. per yard, ready money, but in barter he will have 2s. 3d. E has broadcloth, for which he will have, in barter, 36s. 6d. 3qrs. What ought to be the cash price of E's cloth, and how many yards of ribbon ought D to give him for 488 yards of broadcloth?

Ans. { E's cloth, 32s. 6d.
7930 yards ribbon.

SUPPLEMENT

TO INTEREST, DISCOUNT, BARTER, LOSS AND GAIN.

1. What is the interest of $365,25, for 1 year, 3 months and 2 days? *Ans*, $27,515.

2. What will $1002,153 amount to in 4 years, 1 month and 15 days, at simple interest.? *Ans.*, $1245,175.

3. What is the interest of $125000 for 1 day? *Answer*, $20,833.

4. How much will £300 amount to in 5¾ years, at 3¼ per cent.? *Ans.*, £356 1s. 3d.

5. What is the amount of £10 15s. 6d. for 16 years and 10 months? *Ans.*, 13s. 1d. 3qrs.

6. How much will $185,26 amount to in 2 years, 3 months and 11 days, at 7½ per cent.? *Ans.*, $216,944.

7. How much will $298,59 amount to from May 19th, 1797, to Aug. 11th, 1798, at 8 per cent.? *Ans.*, $327,984.

8. What is the interest of $658, from Jan. 9th, to the 9th of Oct. following, at ½ per cent.? *Ans.*, $2,467.

9. What principal will amount to $1319,90, in 5 years and 8 months? *Ans.*, $985.

10. Took up a note, April 29, 1799, which amounted to $205,86, dated June 14, 1798, on interest at 5¾ per cent. What was the sum borrowed? *Ans.*, $196.

11. A note of 6 years standing amounted to £3810; the principal was £3000: what was the rate per cent.? *Ans.*, 4½.

12. At what rate per cent. will $420 amount to $520,80, in 8 years? *Ans.*, 3 per cent.?

13. At what rate per cent. will £413 12s. 6d. amount to £546 3s. 8d. in 4¾ years? *Ans.*, 6¾.

14. In what time will $500 amount to $725, at $5 per cent.? *Ans.*, 9 years.

15. In what time will a note of £420 amount to £520 16s. at 3 per cent.? *Ans.*, 8 years.

16. What will be the amount of $597,75, in 20 years, at 6 per cent., compound interest? *Ans.*, $1917,077.

17. Gave a note for £450, payable in 3 years, at 5 per cent., compound interest. To what did it amount?
Ans., £520 18s. 7½d.

18. What is the amount of £217, for 2¼ years, at 5 per cent., interest payable quarterly? *Ans.*, £242 13s. 4½d.

19. Bought a quantity of goods to the amount of £250, ready money, and sold them for £300, payable in 9 months. What was the gain in ready money, discounting at 6 per cent.? *Answer*, £47 12s. 8d. 1qr.

20. What is the present worth of $1000, payable one half in 4 months, the other half in 8 months, discounting at the rate of 5 per cent.? *Ans.*, $875,674.

21. How much tea, at 9s. 6d. per pound, must be given in barter for 156 gallons of wine, at 12s. 3½d. per gallon?
Ans., 201 lbs. 13$\frac{27}{57}$oz.

22. A has 240 bushels of rye, at 90 cents per bushel, ready money, which he barters with B, at 95 cents, for wheat which cost 99 cents per bushel. How many bushels of wheat must he receive for his rye, and at what price?
Ans., 218$\frac{2}{11}$ bushels, at $1,04½ per bushel.

23. A and B barter. A has cloth which cost him 28d. B's cost him 22d. B puts his cloth at 25d. in barter. How high must A rate his cloth, to gain 10 per cent. in the trade? *Ans.*, 35d.

24. Bought 100 yards of cloth, at $2 per yard. How must I sell it per yard to gain $50? *Ans.*, $2,50.

25. Bought cloth at $1,50 per yard, which not proving as good as I expected, I am willing to lose 17½ per cent. How must I sell it per yard? *Ans.*, $1,237.

26. Bought 50 gallons of wine at 4s. per gallon. By accident, 10 gallons leaked out. How must I sell the remainder per gallon to gain 10 per cent. upon the whole cost? *Ans.*, 5s. 6d.

27. A man sells a quantity of corn at $1 per bushel, and gains 20 per cent. Some time after, he sold of the same to the amount of $37,50, and gained 50 per cent. How many bushels were there in the last parcel, and at what rate did he sell it per bushel?
Ans., 30 bushels, at $1,25 per bushel.

EQUATION OF PAYMENTS.

Equation of Payments is the method of finding the mean time for the payment of several debts due at different times.

1. If a man owes me $10, to be paid in 4 months, and $5 to be paid in 7 months, and he wishes to pay the whole at once, in what time should the whole be paid?

It is evident, that the use of $10 four months is the same as the use of $1 forty months; and the use of $5 seven months is the same as the use of $1 thirty-five months. Then $10+$5=$15, and 40+35=75 months. Thus it appears, that the use of $10 for four and $5 for seven months, is the same as the use of $1 for seventy-five months: $15, therefore, may be used $\frac{1}{15}$ as long as $1. That is, $\frac{1}{15}$ of seventy-five months, 75÷15=5 months, the answer. Hence the RULE.

Multiply each payment by the time when it becomes due, and divide the sum of the products by the sum of the payments, and the quotient will be the time required.

2. A merchant has owing him $420, to be paid as follows: $100 in 8 months, $100 in 2 months, and $220 in 5 months. In what time ought the whole to be paid at once? *Ans.*, 5 months.

3. A owes B $800, to be paid as follows: $200 in 3 months, $150 in 4 months, and the remainder in 8 months. What is the equated time for the payment of the whole? *Ans.*, 6 months.

4. A owes B $380, to be paid as follows: $100 in 6 months, $120 in 7 months, and $160 in 10 months. What is the equated time for the payment of the whole? *Ans.*, 8 months.

5. A merchant has owing him $698, of which $181 is to be paid at the present time, $199 in 3 months, and $318 in 8 months. What is the equated time for the payment of the whole? *Ans.*, 4½ months.

6. A owes B $500, of which ½ is to be paid in 3 months, ⅕ in 8 months, and the remainder in 2 months. What is the equated time for the payment of the whole? *Ans.*, 5 months, 21 days.

7. A has owing him $924, of which ⅓ is to be paid in 3 months, ½ in 2 years. In what time ought the whole to be paid? *Ans.*, 13 months.

Questions. 1. What is Equation of Payments? 2. Rule?

8. I have three notes against a man; one of $400, due in 5 months; one of $500, due in 6 months; and the other of $350, due in 9 months; and he wishes to pay the whole at once. In what time ought he to pay it?

Answer, 6,52 months.

9. A owes B $960, of which $\frac{1}{2}$ is due in 3 months, $\frac{1}{3}$ in $1\frac{1}{2}$ months, $\frac{1}{6}$ in 9 months. What is the equated time for payment of the whole? *Ans.*, 3 months, 15 days.

10. A merchant bought goods to the amount of $3000, and agreed to pay $500 in cash, $600 in four months, and the remainder in 9 months; but they agree to make one payment of the whole. What is the equated time?

Answer, 6 months, 15 days.

FELLOWSHIP.

FELLOWSHIP is a rule by which merchants and others, trading in company, may ascertain their respective gain or loss, in proportion to each man's share in the joint stock.

The money, or value of property vested in trade, is called the *Capital*, or *Stock*.

The gain or loss to be shared by the company, is called the *Dividend*.

When the several stocks are employed without regard to time, it is called *Single Fellowship*.

1. Two men, A and B, bought a horse for $60, of which sum A paid $40, and B paid $20. They sold the horse for $90. What was each man's share of the gain?

It is evident, that each man's share of the *gain* should bear the same ratio to the *whole gain*, that his *share* of the *stock* bears to the *whole stock*. Now, the whole stock was $60, of which A paid $40; then A paid $\frac{40}{60}=\frac{2}{3}$ of the whole stock, and B paid $\$20=\frac{20}{60}=\frac{1}{3}$ of the whole stock. As the whole gain was $30, A's share is $\frac{2}{3}$ of $30=\$20$, and B's share is $\frac{1}{3}$ of $30=\$10$. Hence the RULE:

As the *whole stock* is to each *man's stock*, so is the *whole gain or loss, to each man's share* of the gain or loss. Or

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QUESTIONS. 1. What is Fellowship? 2. What is capital, or stock? 3. What is the dividend? 4. What is Single Fellowship? 5. What is the rule? 6. What is the method of proof?

the question may be expressed thus: What gains each individual stock, if the whole stock give —— gain?

Whole stock $60—A's stock $40.
Whole gain $30—B's stock $20.

Operation.

How much A's gain.	40$ A's stock.
Whole stock $60	30$ whole gain.
$	20 *Ans.* A's gain.

Operation.

B's gain.	20$ B's stock.
$60	30$
$	10 *Ans.* B's gain.

Proof—Add together the respective gains, and, if the work be right, their sum will equal the whole gain.

2. A, B and C trade in company; A's stock is $240; B's $360, and C's $600; they gain $325. What is each man's share of the gain?

Answer,
A's gain, $ 65.
B's gain, $ 97,50.
C's gain, $162,50.

3. A and B bought a lot of land for $1280, of which B paid $400, and A the remainder; they sold it so as to gain $200. What was each man's share of the gain?

Answer,
A's gain, $137,50.
B's gain, $ 62,50.

4. A and B owned a ship valued at $72000—lost at sea. insurance $50,000; what was each man's loss, supposing A owned 3 times as much as B?

Answer,
A's loss, $16500.
B's loss, $ 5500.

5. A man dying, leaves property to the amount of $3000. A has a note of $600 against the estate, B has a note of $1800, and C a note of $1600. How much must each lose?

Answer,
A's loss, $150.
B's loss, $450.
C's loss, $400.

6. Three partners, A, B and C, shipped 216 horses for the south. A's share of the cost of the horses was $2880; B's, $5760; C's, $4320. During the voyage they were obliged to throw 90 overboard. How many horses did each partner lose?

Answer,
A lost 20.
B lost 40.
C lost 30.

7. A and B trade in company. A's stock was 60 guineas, and his share of the gain was $\frac{5}{8}$. What was B's stock?

Answer, 36 guineas.

8. Three men gained in an adventure $96. A put in a certain sum, B put in twice as much as A, and C as much as A and B both. What was each man's share of the gain?

Answer, A's $16.
B's $32.
C's $48.

9. Two men trade in company; their joint stock is $800, of which B put in $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{6}{8}$ of 4 times the whole. What is each man's stock?

Answer, A's $200.
B's $600.

10. A, B and C trade in company. A's stock is $250, B's $300, C's $550; they lose 5 per cent. by trading. What is each man's share of the loss?

Answer, A's loss $12,50.
B's loss $15,00.
C's loss $27,50.

11. A man by his will left his estate to his children, as follows: to A he gave $5000, to B $4500, to C $4500, and to D $4000; but his whole estate amounted to but $12000. How much did each receive?

Answer, A received $3333,33$\frac{1}{3}$.
B " $3000,00.
C " $3000,00.
D " $2666,66$\frac{2}{3}$.

ASSESSMENT OF TAXES.

In order to the assessment of taxes on a town, the following facts should be known:

I. The amount of tax assessed by the Legislature of the State.

II. The inventory of all the rateable property in the town.

RULE.

I. From the inventory of the *real* and *personal* property of the whole town, deduct the amount of poll taxes.

II. Find the tax on a dollar, and multiply each man's inventory by it, and to the product add his poll tax.

QUESTIONS. 1. What facts should be known, in order to the assessment of taxes? 2. What is the rule?

EXAMPLES.

1. A town inventoried at $160,000, raises a tax of $3400. There are 400 rateable polls, taxed 50 cents each. What is the tax on a dollar, and what is A's tax, whose *real* and *personal* estate is inventoried at $1683, and who pays for one poll?

First deduct the amount of poll tax for 400 polls, at 50 cents each, 400×,50=$200, amount of poll tax; then $3400—200=$3200 to be assessed on the whole property. Secondly, find the tax on a dollar.

Operation.

What tax. | 1 $
$160,000 | 3200 $

Answer, | ,02 cts. on $1.

Then to find A's tax, multiply the amount of his inventory by the tax on a dollar, and to the product add his poll tax—thus, $1683×,02=$33,66+,50=$34,16, A's tax.

Or, having found the tax on a dollar, a table may be formed, containing the tax on 1, 2, 3, or to 20 dollars; then on 30, 40, &c., to 100 dollars; then on 110, 120, &c., to 1000 dollars. Then, having the inventory of the property of an individual, his tax may be readily made out.

TABLE.

Tax on $1 is	$,02	Tax on $16 is	$,32	Tax on $130 is	$2,60
- - 2 -	,04	- - 17 -	,34	- - 140	2,80
- - 3 -	,06	- - 18 -	,36	- - 150	3,00
- - 4 -	,08	- - 19 -	,38	- - 160	3,20
- - 5 -	,10	- - 20 -	,40	- - 170	3,40
- - 6 -	,12	- - 30 -	,60	- - 180	3,60
- - 7 -	,14	- - 40 -	,80	- - 190	3,80
- - 8 -	,16	- - 50 -	1,00	- - 200	4,00
- - 9 -	,18	- - 60 -	1,20	- - 300	6,00
- - 10 -	,20	- - 70 -	1,40	- - 400	8,00
- - 11 -	,22	- - 80 -	1,60	- - 500	10,00
- - 12 -	,24	- - 90 -	1,80	- - 600	12,00
- - 13 -	,26	- - 100 -	2,00	- - 700	14,00
- - 14 -	,28	- - 110 -	2,20	- - 800	16,00
- - 15 -	,30	- - 120 -	2,40	- - 900	18,00
				- - 1000	20,00

We find by the table,

the tax on	\$1000	to be	\$20,00
the tax on	600	‘ ‘	12,00
‘ ‘ on	80	‘ ‘	1,60
‘ ‘ on	3	‘ ‘	,06
A's inventory	\$1683		\$33,66
		Poll tax,	,50
		Amount of A's tax,	\$34,16

2. The inventory of *real* and *personal* estate in the town of ——, for the year 1837, is \$800,000. The amount assessed on each rateable poll is \$1. The number of polls is 400. The amount of town tax voted to be raised for the year 1837, is \$2000. The proportion of State tax for said town for that year, is \$400—County tax \$200—the amount of school tax is \$800—the highway tax is \$1200. How much is A's Town, State, County, School and Highway tax, whose whole estate is inventoried at \$5000, and who pays for one poll?

	Town tax.	*State.*	*County.*	*School.*	*Highway.*	*Total.*
Answer,						

DOUBLE FELLOWSHIP.

1. Two men, A and B, hire a pasture for \$36. A put in 8 oxen 6 weeks, and B 12 oxen 8 weeks. How much must each pay?

It is evident that the pasturage of 8 oxen 6 weeks is the same as of 1 ox 48 weeks; and the pasturage of 12 oxen 8 weeks, is the same of 1 ox 96 weeks. The shares of A and B are the same as though A had put in 1 ox 48 weeks, and B 1 ox 96 weeks—$96+48=144$ weeks. Then A's share of the rent will be $\frac{48}{144}=\frac{1}{3}$ of \$36=\$12, and B's share will be $\frac{96}{144}=\frac{2}{3}$ of \$36=24. Hence the RULE:

Multiply each man's stock by the time it is continued in trade, and consider the product his share of the joint stock, and proceed as in Single Fellowship.

☞ *For Operation, see next page.*

Operation.

How many $	48 weeks.
Weeks 144	36 $
$	12 A's share.

Operation.

How many $	96 weeks.
Weeks 144	36 $
$	24 B's share.

2. A and B trade in company. A put in $3000 for 6 months, B put in $4000 for 10 months, and C put in $2500 for 12 months; they gained $880. What is each man's share of the gain?

Answer, A's share $180.
B's share $400.
C's share $300.

3. Three men trade in company. A put in $4000 for 12 months, B put in $3000 for 15 months, and C put in $5000 for 6 months; the whole gain was $615. What was their respective shares?

Answer, A's gain $240.
B's gain $225.
C's gain $150.

4. A, B and C made a stock for 2 years. A put in at first $1000. At the end of 6 months he put in $500 more. B put in $1600, and after 8 months took out $400. C put in $2000 for 20 months, and then took out $1500. They gain $1000. What is each man's share?

5. A, B and C lost in trade $263,90. A's stock was $580 for 6½ months; B's stock was $580 for 9½ months; C's stock was $870 for 8⅔ months. What is each man's share of the loss?

Answer, A's loss $ 59,15.
B's loss $ 89,45.
C's loss $118,30.

6. A commenced business on the first of January, with a capital of $3800; on the first of May, he took in B as a partner, with a capital of $2700; on the first of August, they admit C as a partner, with a capital of $4000; at the end of the year they dissolve partnership; each took his share of the stock and gain, the gain being $4360. How much did each take?

Answer, A took $6080.
B " 3780.
C " 5000.

INVOLUTION.

Involution is multiplying a number into itself. The product is called a power; the number so multiplied, is called a *root*, or the *first* power. The product of any number multiplied into itself, is called the *second* power, or *square*. If the square be multiplied by the *first* power, the product is called the *cube*, or *third* power.

The power is sometimes denoted by a small figure, called the *index* or *exponent* of the power, placed above the given number at the right hand—Thus, 3^2 denotes that the second power of 3 is required, or it shows how many times 3 is to be involved or multiplied. This may be illustrated by the following:

$2^2=2\times2=4$, the second power of 2.

$2^3=2\times2\times2=8$, the third power of 2.

$2^4=2\times2\times2\times2=16$, the fourth power of 2.

$2^5=2\times2\times2\times2\times2=32$, the fifth power, or sursolid.

$2^6=2\times2\times2\times2\times2\times2=64$, the 6th power,or square cubes.

The product of any two powers is always that power whose index is the sum of the indices, or exponents, of the power multiplied thus—

0	1	2	3	4	5	6	7	8	9	10
1	2	4	8	16	32	64	128	256	512	1024

If 16, which is the fourth power of 2, be multiplied into 64, the 6th power of 2, we shall have 1024, the power indicated by the multiplication of $2^4\times2^6=2^{6+4}=2^{10}=1024$.

EXAMPLES.

1. What is the square, or second power, of 25?
Answer, 625.

2. What is the square, or second power, of 145?
Answer, 21025.

3. What is the cube, or third power, of 23?
Answer, 12167.

4. What is the cube, or third power, of 159?
Answer, 4019679.

Questions. 1. What is Involution? 2. What is a power? 3. What is a root? 4. What is the second power?—the third power? 5. How is a power denoted?

5. What is the biquadrate, or fourth power, of 29?
Answer, 707281.

6. What is the fifth power of 134?

7. What is the sixth power of 225?

8. What is the square of 1? 9. What is the cube of 1?
Answer, 1—1.

10. What is the square of $\frac{1}{2}$? 11. What is the cube of $\frac{1}{2}$?
Answer, $\frac{1}{4}$—$\frac{1}{8}$.

12. What is the cube of 1,5? 13. What is the cube of 2,25?

14. What is the square of $2\frac{2}{5}$?

Note. Mixed numbers may be reduced to improper fractions, before involving: Thus $2\frac{2}{5}=\frac{12}{5}$; or they may be reduced to decimal: Thus: $2\frac{2}{5}=2{,}4$.

Answer, $\frac{144}{25}$, 5,76.

The powers of the nine digits, from the first to the ninth, may be seen by the following

TABLE.

Roots ——	1	2	3	4	5	6	7	8	9
Squares ——	1	4	9	16	25	36	49	64	81
Cubes ——	1	8	27	64	125	216	343	512	729
4th power-	1	16	81	256	625	1296	2401	4096	6561
5th power-	1	32	243	1024	3125	7776	16807	32768	59049
6th power-	1	64	729	4096	15625	46656	117649	262144	531441
7th power-	1	128	2187	16384	78125	279936	823543	2097152	4782969
8th power-	1	256	6561	65536	390625	1679616	5764801	16777216	43046721
9th power-	1	512	19683	262144	1953125	10077696	40253607	134217728	387420489

EVOLUTION.

Evolution, the opposite of Involution, is the extracting of the root of any number, or the finding of such a number as, when multiplied into itself a certain number of times will produce a given number. Thus, 3 is the square root of 9, because $3\times3=9$; also, 3 is the cube root of 27, because $3\times3\times3=27$.

Any given power may be found by a continued multiplication of the number into itself; yet there are numbers whose precise root can never be found: but, by the use of decimals, we can arrive sufficiently near for all practical purposes.

Questions. 1. What is Evolution? 2. How may any given power be found? 3. Can the precise roots of all powers be found? 4. How can we approximate sufficiently near for practical purposes? 5. What is a number called whose precise root cannot be found?

A number whose precise root cannot be found, is called a *surd*, or *irrational* number, and its root a *surd* root.

The square root may be denoted by this character, $\surd$, called the *radical sign*, placed before the power; and the other roots by the same sign, with the index of the root placed over it, or by the fractional indices placed on the right hand. Thus the square root of 9 is expressed, $\surd 9$, or $9^{\frac{1}{2}}$, and the cube root of 27 thus: $\sqrt[3]{27}$, or $27^{\frac{1}{3}}$.

The method of denoting roots by the fractional indices is preferable, as, by it, not the root only is denoted, but the power. The numerator of the index denotes the power, and the denominator the root of the number over which it is placed.

If the power is expressed by several numbers, with the sign, +, or —, between them, a line, or *vinculum*, is drawn from the top of the sign over all the numbers. Thus, the square root of 12+4 is $\sqrt{12+4}=4$, and the cube root of 359—14 is $\sqrt[3]{359-14}=7$.

EXTRACTION OF THE SQUARE ROOT.

To extract the square root is to find a number, which, multiplied into itself, will produce a given number; or, it is to find the length of one side of a square surface, whose area is expressed by the given number.

1. What is the length of one side of a square garden, containing 576 square rods; or what is the square root of 576?

* We first distinguish the number whose root is to be found, into periods of two figures each, denoted by the in-

---

* It is distinguished into periods of two figures each, because the second power can never have more than twice as many figures as its root, and never but one less than twice as many. The third power can never have more than three times as many figures as its root, and never but two less than three times as many. Distinguish, therefore, any number into periods of as many figures as are denoted by the index of the root.

QUESTIONS. 1. What is the advantage of denoting roots by the fractional indices? 2. What is it to extract the square root? 3. Give the illustration of the first example.

dex of the root. By the number of periods, we perceive that the root will consist of two figures, a unit and ten. As the second power of ten cannot be less than a hundred, we look for the square of tens, in the second, or left hand period, which is 5. We find the nearest square in 5 to be 4, and its root 2, or 2 tens, which we place in the quotient as the first figure of the root; and its square, 4, or 400, under the period, and subtracting it, we have a remainder of 1, or 100, to which we add 76, the next period. Had the garden contained but 400 square rods, we should now have obtained the length of one side, 2 tens =20, and 20×20=400; consequently, 400 rods would be disposed of in the form of a square.—(See Fig. I.) But we have a remainder of 176 rods to be added to the square, and in such a manner that its form shall not be altered. We must, therefore, make an equal addition on two sides. Then 20+20=40, the length of the whole addition. To find the width of the addition, we place the double of the root already found, on the left hand of the dividend, for a divisor. If we divide 176, the number of rods to be added, by 40, the length of the addition, (or 17 by 4, rejecting the unit figure of the dividend and divisor,) we have 4 rods, the width of the addition; then 40×40=160, the number of rods added on the two sides; still there is a remainder of 16 rods. As the additions

$\dot{5}7\dot{6}(2$
4
——
176

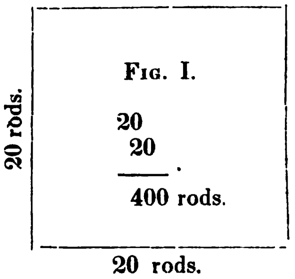
Fig. I.

$\dot{5}7\dot{6}(24$
——
44)176
176
——
Proof: 24×24=576.

made are no longer than the sides of the square, there will be a deficiency in the corner, (see Fig. II.) of a square, whose sides are equal to the width of the addition, 4×4=16 rods. We, therefore, place 4, the last quotient figure, on the right of the divisor, because its square is necessary to supply this deficiency. The whole divisor, now multiplied by the last quotient figure, equals 176, the number of rods which were to be added to the square. We have now obtained 24, the root of 576, or the length of one side of a square garden, containing 576 square rods. Proof by Involution: 24×24=576.

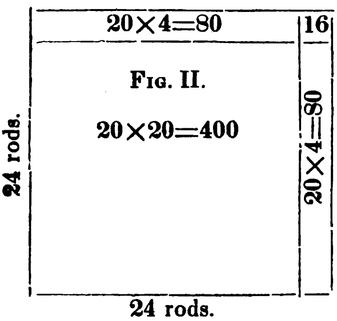

Fig. II.

From the preceding example and illustration we derive the following

RULE.

I. Distinguish the given number into periods of two figures each, by putting a dot over the units, and another over the hundreds, and so on. The dots show the number of figures of which the root will consist.

II. Find the root of the greatest square number in the left hand period, and place it as a quotient in division. Place the square of the root found, under said period, and subtract it therefrom, and to the remainder bring down the next period for a dividend.

III. Double the root already found, for a divisor; see how often the divisor is contained in the dividend, (excepting the right hand figure) and place the result for the next figure in the root, and also on the right hand of the divisor.

IV. Multiply the divisor by the figure in the root last found, and subtract the product from the dividend; to the remainder, bring down the next period for a new dividend. Double the root now found, for a new divisor, and proceed in the operation as before, until all the periods are brought down.

Note. Doubling the right hand figure of the last divisor, observ-

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Questions. 4. Repeat the rule, and give the reason for each step.

ing to add 1 to the place of tens, when the double of the unit figure is over ten, is the same as doubling the root, or quotient.

EXAMPLES.

2. What is the square root of 119716?

Operation.

```
 .  .  .
119716(346, Answer.
 9
------
64)297
   256
-------
686)4116
    4116
```

3. What is the square root of 1444? *Ans.* 38.

4. What is the square root of 59536? *Ans.*, 244.

5. What is the square root of 124896? *Ans.*, 353,4+.

NOTE. When there is a remainder, after all the figures are brought down, ciphers may be annexed, and the operation continued to any assigned degree of exactness.

6. What is the square root of 67321? *Answer*, 259,46+.

7. What is the square root of 25289. *Answer*, 159,02+.

8. What is the square root of 21027. *Answer*, 145,006+.

9. What is the square root of 6842,723400? *Answer*, 82,7177+.

NOTE. When there are whole numbers and decimals in the given sum, point off both ways from the units' place; if the decimals be an odd number, annex ciphers, and make them even.

10. What is the square root of 10,4976? *Answer*, 3,24.

11. What is the square root of 336,234? *Answer*, 18,333+.

12. What is the square root of ,108241? *Answer*, ,329.

13. What is the length of a square field containing 7744 square rods? *Answer*, 88 rods.

14. What is the square root of $\frac{9}{81}$? *Answer*, $\frac{3}{9}$.

NOTE. The square root of a fraction may be found by extracting the root of the numerator and denominator.

15. What is the square root of $\frac{16}{25}$? *Answer*, $\frac{4}{5}$.
16. What is the square root of $\frac{4}{49}$? *Answer*, $\frac{2}{7}$.
17. What is the square root of $\frac{49}{64}$? *Answer*, $\frac{7}{8}$.
18. What is the square root of $\frac{1}{2}$? *Answer*, ,223+.

Note. When the numerator and denominator are *surd* numbers, reduce the fraction to a decimal, and extract the root as above directed.

19. What is the square root of $\frac{3}{4}$? *Answer*, ,866+.

20. What is the square root of $\frac{7}{8}$? *Answer*, ,2958+.

21. How many rows on one side of a square corn-field, containing 15376 hills? *Answer*, 124.

22. An army of 242064 men are drawn up in a solid body, in the form of a square. What is the number of men in rank and file? *Answer*, 492.

23. A man has 841 peach trees, which he wishes to plant in the form of a square. How many must be planted in each row? *Answer*, 29.

24. There is a circular pond, containing 110889 square rods; what will be the length of a square field containing the same number of rods? *Answer*, 333 rods.

25. A number of men gave £22 1s. for a charitable purpose, each giving as many shillings as there were men. What was the number of men? *Answer*, 21.

26. What is the width of a parallelogram, containing 338 square rods, whose length is twice the width? *Answer*, 13 rods.

27. What is the length of one side of a square acre of land? *Answer*, 12,64+.

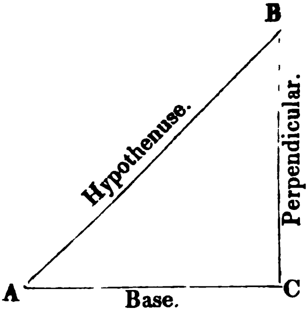
28. The diameter of a circle is 6 inches. What is the diameter of a circle 4 times as large? *Answer*, 12.

Note. Circles are to one another as the squares of their diameters; therefore, to find the required diameter, square the *given* diameter, multiply the square by the given ratio, and the square root of the product will be the diameter required.

29. The diameter of a circle is 24 feet. What is the diameter of a circle one fourth as large? *Ans.*, 12 feet.

30. In the right angled triangle A, B, C, the side A, C is 9 feet, and the side B, C, 12 feet. What is the length of the side A, B? ☞ *See next page.*

Questions. 1. How are circles to each other? 2. Having the hypothenuse, and one other side of a triangle given, how is the remaining side found? 3. Rule for finding the hypothenuse, having the other two sides given?



In every right angled triangle, the square of the hypothenuse is equal to the sum of the squares of the base and perpendicular; therefore, the square root of the sum of the squares of the base and perpendicular, will be the hypothenuse, and the square root of the difference of the square of the hypothenuse, and either of the other sides, will be the remaining side.

$$AC^2=9^2=81$$
$$BC^2=12^2=144$$
$$AB^2=\overline{225}$$
$$AB=\sqrt{225}=15 \text{ ft.}$$ *Answer.*

31. What is the distance between the opposite corners of a room, 20 feet in length and 15 in width? *Ans.*, 25 feet.

32. If the distance between the opposite corners of a room be 25 feet, and the width of the room be 15 feet, what is the length? *Ans.*, 20 feet.

33. If a room be 20 feet in length, and 25 feet between the opposite corners, what is the width? *Ans.*, 15 feet.

34. Two men owning a pasture, 32 rods in width, and 50 rods between the opposite corners, agreed to divide said pasture into two equal parts by a wall running through it lengthwise. Suppose they pay 50 cents a rod for building the wall, what does it cost them? *Ans.*, $19,209.

35. Suppose a ladder, 50 feet long, to be so placed as to reach a widow 30 feet from the ground on one side of the street, and without moving it at the foot, will reach a window 20 feet high, on the other side, what is the width of the street? *Ans.*, 85,825 feet.

36. Two men travel from the same place—one due east, the other due north. One travels 40 miles the first day, the other 30. What is the nearest distance between them at night? *Ans.*, 50 miles.

37. A and B travel in the same direction. They are 20 miles apart when they set out, and maintain the same distance through the day. A travels 45 miles, and B 25. What is the distance between them at night?

Ans., 32+ miles.

38. Suppose a pine tree to stand 25 feet from the end of a house 40 feet in length, the foot of the tree being on a level with the foundation of the chimney, which stands in the centre of the house, and a line reaching from the foot of the tree to the top of the chimney be 75 feet, what is the height of the chimney? and if the height of the tree be $\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{4}{7}$ of 14 of the height of the chimney, what will be the length of a line reaching from the top of the chimney to the top of the tree?

Ans., { 60 feet, height of the chimney.
75 feet, length of the line.

To find a mean proportional between two numbers.

RULE.

Multiply the given numbers together, and the square root of their product is the mean proportion sought.

1. What is the mean proportional between 3 and 12?

Operation.

$3 \times 12 = 36$, and $\sqrt{36} = 6$ *Answer.*

It is evident, that the ratio of 3 to 6 is the same as the ratio of 6 to 12, for $\frac{6}{3} = 2$, and $\frac{12}{6} = 2$.

2. What is the mean proportional between 12 and 48.

Ans., 24.

3. What is the mean proportional between 9 and 81?

Ans., 27.

4. What is the mean proportional between 25 and 625?

Ans., 125.

EXTRACTION OF THE CUBE ROOT.

A CUBE is a solid body, having six equal sides. A cube root is the length of one of those sides.

To extract the cube root, is to find a number which,

QUESTIONS. 1. Rule for finding a mean proportional between two numbers? 2. What is a cube? 3. What is a cube root? 4. What is it to extract the cube root?

multiplied into its square, will produce the given number; or, it is to find the length of one side of a solid body whose content the given number expresses.

1. What is the length of one side of a solid block containing 13824 solid inches, or what is the cube root of 13824?

Having distinguished the given number into periods of three figures each, denoted by the index of the root, we perceive, by the number of periods, that the root will consist of two figures. As the cube of ten cannot be less than a thousand, $10\times10\times10=1000$, we look for the cube of tens in the second, or left hand period. We find, by trial, the greatest cube in 13 or 13000, to be 8 or 8000, and its root 2, or 2 tens, which we place in the quotient, as the first figure of the root, and its cube, $20\times20\times20=8000$, under that period; and subtracting it, we have a remainder of 5, or 5000—to which we bringdown the next period. Had the cube contained but 8000 solid inches, we should now have found its root, or the length of one side. But we have 5824 inches to be added to the cube, and in such a manner that its cubic form shall not be altered. It is obvious, that an equal addition must be made on three sides. As each side is 20 inches square, we have $20\times20\times3=1200$; or, which is the same thing, multiply the square of the quotient by 300. $2\times2\times300=1200$ inches surface, to which the additions are to be made. It will be seen that there are three deficiences along the sides where the additions meet, 20 inches in length, $20\times3=60$, or multiply the quotient by 30; $2\times30=60$. We have, then, $1200+60=1260$, which may be considered the points where the additions are to be made. Then $5824\div1260=4$ inches, the thickness of the addition, or the second figure of the root. The area of the sides multiplied by the thickness, $1200\times4=4800$ inches, the amount of the addition upon the sides.

Operation.

$1\dot{3}82\dot{4}(24$, root.

$2^3=2\times2\times2=\ 8$

$2^2\times300+60=1268)\ 5824$

$1200\times4=4800$

$60\times4\times4=\ 960$

$4\times4\times4=\ \ 64$

5824

Then the number of inches necessary to fill the deficiencies where the additions on the sides meet, is 60×4×4=960 inches. Still there is a deficiency of a small cube, whose dimensions are equal to the thickness of the additions: 4×4×4=64 inches. This supplied, and the cube is completed. The sum of all the additions will be a subtrahend equal to the dividend; 4800+960+64=5824. We have now found the length of one side of the cube to be 24 inches. Proof by Involution; 24×24×24=13824.

The foregoing operation can be better illustrated by blocks prepared for the purpose. It is necessary to have one cubical block, of a convenient size to represent the greatest cube in the left hand period, and three other blocks, equal to the sides of the first block, but of indefinite thickness, to represent the additions upon the sides. Then three other blocks, equal in length to the sides of the cube, and their other dimensions equal to the thickness of the additions, on the sides of the cube. Lastly: a small cubic block, of dimensions equal to the thickness of the additions, to fill the deficiency at the corner. By placing these blocks as above described, the several steps in the operation may be easily understood.

From the foregoing example and illustration we derive the following RULE.

I. Distinguish the given number into periods of three figures each, beginning at the right hand.

II. Find the greatest cube in the left hand period, and place its root as a quotient in division.

III. Subtract the cube from said period, and to the remainder bring down the next period for a dividend.

IV. Multiply the square of the quotient by 300, calling it the triple square, and the quotient by 30, calling it the triple quotient, and the sum of these call the divisor.

V. Seek how many times the divisor is contained in the dividend, and place the result in the quotient, for the second figure of the root.

VI. Multiply the triple square by the last quotient figure,

QUESTIONS. 5. What is the rule? 6. Why do you distinguish the given number into periods of three figures each? 7. Why do you multiply the square of the quotient by 300? 8. Why the quotient by 30? 9. Why the triple square by the last quotient figure? 10. Why the triple quotient by the square of the last quotient figure? 11. Why do you add to these the cube of the last quotient figure? 12. Explain the process of illustrating this rule by blocks.

and write the product under the dividend; multiply the triple quotient by the square of the last quotient figure, and place this product under the last; under these write the cube of the last quotient figure, and call their sum the subtrahend. Subtract the subtrahend from the dividend, and to the remainder bring down the next period, for a new dividend, and proceed as before, till the work is finished.

EXAMPLES.

2. What is the cube root of 1906624?

Operation.

$$\begin{array}{r|l} 1\times1\times300=300 & \dot{1}90\dot{6}62\dot{4}(124\ \textit{Answer.} \\ 1\times\ 30=\ 30 & 1 \\ \hline \text{Divisor, }\ 330 & 906\text{, dividend.} \end{array}$$

$$\begin{array}{r} 300\times2=600 \\ 30\times2^2=120 \\ 2^3=\ \ \ 8 \\ \hline 728\text{, subtrahend.} \end{array}$$

$$12^2\times300+12\times30=43560)178624$$

$$\begin{array}{r} 43200\times4=172800 \\ 360\times4^2=\ \ \ 5760 \\ 4^3=\ \ \ \ \ \ 64 \\ \hline \text{Subtrahend, }\ 178624 \end{array}$$

3. What is the cube root of 941192? *Answer*, 98.
4. What is the cube root of 6331625? *Answer*, 185.
5. What is the cube root of 11543176000? *Answer*, 2260.
6. What is the cube root of 34,328125? *Ans.*, 3,25.
7. What is the cube root of ,000729? *Answer*, ,09.
8. What is the cube root of ,003375? *Answer*, 15.
9. What is the cube root of 5? of 3?
10. What is the cube root of $\frac{8}{125}$? *Answer*, $\frac{2}{5}$.
11. What is the cube root of $\frac{216}{343}$? *Answer*, $\frac{6}{7}$.
12. What is the cube root of $\frac{1728}{2197}$? *Answer*, $\frac{12}{13}$.

13. What is the cube root of $\frac{1}{10}$?

14. What is the cube root of $\frac{1}{27}$? *Answer*, $\frac{1}{3}$.

15. A certain hill contains 11543176 cubical feet. What is the length of one side of a cubical mound, containing an equal number of feet? *Answer*, 226 feet.

16. The contents of an oblong cellar is 9261 cubical feet. What the length of one side of a cubical cellar of the same capacity? *Answer*, 21 feet.

17. A merchant bought cloth to the amount of $393,04, but forgets the number of pieces, and also the number of yards in each piece, and what the cloth cost per yard; but remembers that he paid as many cents per yard as there were yards in each piece, and that there were as many in each piece as there were pieces. What did he pay per yard? *Answer*, 34 cents.

18. What is the width of a cubical vessel, containing 75 wine gallons, each 231 cubic inches?

19. Required the side of a cubic box that shall contain a bushel? *Answer*, 12,9+ inches.

◆

Solids of the same form are to one another as the cubes of their similar sides or diameters.

EXAMPLES.

1. If a bullet, weighing 72 lbs. be 8 inches in diameter, what is the diameter of a bullet weighing 9 lbs.?

Answer, 64 inches.

Statement.

$8^3=512$ 72:9::512:64 or thus:

$$\begin{array}{r|l} & \cancel{9} \\ \cancel{8}\ \cancel{72} & \cancel{512}\ 64 \\ \hline & 6\ \textit{Answer.} \end{array}$$

2. A bullet, 2 inches in diameter, weighs 4 lbs.; what is the weight of a bullet 5 inches in diameter?

Answer, 62½ lbs.

3. If a silver ball, 9 inches in diameter, be worth $400, what is the worth of another ball 12 inches in diameter?

Answer, $948,148+.

QUESTIONS. 13. What proportion have solids to one another?

To find two mean proportionals between two numbers.

RULE.

Divide the greater by the less, and extract the cube root of the quotient; multiply the lesser number by this root, and the product will be the lesser mean; multiply this mean by the same root, and the product will be the greater mean.

EXAMPLES.

1. What are the two mean proportionals between 4 and 256?

$256 \div 4 = 64$ then $\sqrt[3]{64} = 4$ and $4 \times 4 = 16$, the lesser, and $16 \times 4 = 64$, the greater. *Proof*, 4:16::64:256.

2. What are the two mean proportionals between 5 and 625? *Answer*, 25 and 125.

3. What are the two mean proportionals between 7 and 2401? *Answer*, 49 and 243.

◆

EXTRACTION OF ROOTS IN GENERAL.

RULE.

I. Point the given number into periods of as many figures as the index of the root directs—Thus, for the square root, two figures; cube root, three; the fourth root, four, &c.

II. Find by trial the greatest root in the left hand period, and subtract its power from that period, and to the remainder bring down the first figure of the next period, for a dividend.

III. Involve the root already found, to the next inferior power to that which is given, and multiply it by the number denoting the given power, for a divisor, by which find the second figure of the root.

IV. Involve the whole root now found to the given power; subtract it from the given number as before, and bring down the first figure of the next period to the remainder, for a new dividend, and proceed as before, till the work is finished.

QUESTIONS. 14. Rule for finding two mean proportionals between two numbers? 15. Rule for extracting roots in general?

Note. The roots of most of the powers, may be found by repeated extractions of the square and cube root—Thus:

For the 4th root, take the square root of the square root.
For the 6th " take the square root of the cube root.
For the 8th " take the square root of the 4th root.
For the 9th " take the cube root of the cube root.
For the 12th root, take the cube root of the 4th root.

EXAMPLES.

1. What is the square root of 7569?

Operation.

$7\dot{5}6\dot{9}$(87
8×8= 64=square, or 2d power, of the quotient.
8×2=16)116=dividend.
87×87= 7569=square of the quotient.
0000

2. What is the fifth root of 4084101?

Operation.

$4\dot{0}8410\dot{1}$(21
2×2×2×2×2= 32 = 5th power of the quotient.
2×2×2×2×5=80)88 = 1st dividend.
4084101
21×21×21×21×21=4084101=5th power of the quotient.

3. What is the fourth root of 140283207936?

Answer, 612.

4. What is the seventh root of 4586471424?

Answer, 24.

5. What is the ninth root of 1352605460594688?

Answer, 48.

ARITHMETICAL PROGRESSION.

Arithmetical Progression is when a series of numbers increases by a common excess, or decreases by a common difference.

Questions. 1. What is Arithmetical Progression?

When numbers increase by a common excess, they form the *ascending series*, as 2, 4, 6, 8, 10, 12, &c.

When numbers decrease by a common difference, they form the *descending series*, as 12, 10, 8, 6, 4, 2, &c.

The numbers forming the series, are called the terms; the first and last terms are called the *extremes*, and the other terms the *means*.

When any even number of terms differs by Arithmetical Progression, the sum of the two extremes will equal the sum of any two means equally distant from the extremes; as 2, 4, 6, 8, 10, 12. The two extremes, $2\times12=6+8$ the two means. When the number of terms is odd, the double of the mean will equal the sum of the two extremes, or the sum of any two numbers equally distant from the extremes; as 1, 2, 3, 4, 5. The double of the mean $3+2=5+1=6$.

In Arithmetical Progression, five things are to be considered, viz: the first and last terms, the number, common difference, and sum of all the terms; any three of which being given, the other two may be found.

1. If I buy 4 books, giving 2 cents for the first, 4 for the second, and so on, with a common difference of 2, what do I pay for the last book?

It is evident, that if we add 2 cents, the common difference, to the price of the first book, we shall have the price of the second, and so on to the last; thus, $2+2=4$, $4+2=6$, $6+2=8$ cents, the answer. It will be seen that 2, the common difference, is added to every term but the last. If, then, we multiply the number of terms less 1, by the common difference, we have the difference between the cost of the first book and the last; thus, $3\times2=6$, and $6+2=8$, as before. Therefore,

When the first term, the number of terms, and common difference are given, to find the last term:

RULE.

Multiply the number of terms less 1, by the common difference, and to the product add the first term, and the sum will be the common difference.

QUESTIONS. 2. When is the series ascending? 3. When descending? 4. What is meant by the terms? 5. What is meant by the extremes? 6. By the means?

2. If the first term of a series be 5, the number of terms 35, and the common difference 3, what is the last term?
Answer, 35—1×3=102+5=107.

3. If I buy 80 yards of cloth, giving 6 cents for the first, 10 for the second, and so on, with a common difference of 4, what do I pay for the last yard? *Answer*, 322 cts.

4. Suppose a man purchase 40 sheep, paying 3 pence for the first, 10 for the second, and so on with a common difference of 7, what does he pay for the last sheep?
Answer, 276 pence.

5. If 96 acres of land be sold at the rate of 10 cents for the first acre, 19 for the second, and so on, with a common difference of 9, for how much is the last acre sold?
Answer, 865 cents.

6. If I buy 4 books, the prices of which are in Arithmetical Progression, giving 2 cents for the first, and 8 for the last, what is the common difference in the prices of the books?

This question is the reverse of question 1st. 8—2=6, 6÷3=2, the common difference. It is plain, that the difference between the price of the first and last book, is the whole addition made to the price of the first book; and as the addition is made equally to the three books, it is equally plain that the whole addition divided by the number of additions, will be the addition made to the price of each book. Therefore—

When the extremes and number of terms are given, to find the common difference, we have this RULE:

Divide the difference of the extremes, by the number of terms less 1, and the quotient will be the common difference.

7. If the first term of a series be 3, the last term 276, and the number of terms 40, what is the common difference? *Answer*, 7.

8. A man on a journey, travels the first day 2 miles, and increases his travel daily by an equal excess for 15 days, so that the last day he travels 72 miles. What was the daily increase? *Answer*, 5 miles.

9. Bought books, paying 2 cents for the first, and 8 cts. for the last, with a common difference of 2. What number of books did I buy?

As the difference of the extremes divided by the number of terms less 1, will give the common difference, it is evident, that the difference of the extremes, divided by the common difference, will give the number of terms less 1. Then 8—2=6, the difference of the extremes, and 6÷2 =3, which is one less than the number of terms; then 3+1=4, the number of books purchased. Therefore—

When the first and last terms, and the common difference are given, to find the number of terms:

RULE.

Divide the difference of the extremes by the common difference, and the quotient will be 1 less than the number of terms.

10. If the first term of a series be 2, and the last term 72, the common difference 5, what is the number of terms? *Answer*, 15.

11. A man bought sheep, paying at the rate of 3 pence for the first, and 276 for the last, with a common difference of 7. What number did he buy? *Answer*, 40.

12. A man had a number of sons, the common difference of whose ages is 4 years; the youngest is 8, the eldest 40 years old; how many sons had he? *Answer*, 9.

13. If I buy 4 books, paying 2 cents for the first, and 8 cents for the last, how many cents do I pay in all?

If the price of the first book is 2 cents, and the price of the last is 8 cents, it is evident that the average price of the books is half way between 2 cents, the price of the first, and 8 cents, the price of the last book, 2+8÷2=5; then 5, the average price, multiplied by the number of books will give the whole cost; 5×4=20 cents. Therefore—

When the first and last terms, and the number of terms are given, to find the sum of the series.

RULE.

Multiply half the sum of the extremes by the number of terms; the product will be the sum of the series.

14. A man has 9 sons; the youngest is 8, the eldest 40 years old. What is the sum of their ages?
Answer, 216 years.

15. How many times will a clock strike in a day, if constructed like the clocks of Venice, to run till 24 o'clock?
Answer, 300.

16. If a triangular piece of land be 1 rod wide at one end, and 60 at the other, what number of square rods does it contain? *Answer*, 1830.

GEOMETRICAL PROGRESSION.

A GEOMETRICAL PROGRESSION is series of terms, which increase by a uniform multiplier, or decrease by a uniform divisor, as 3, 6, 12, 24, &c., increasing by a uniform multiplier, 2; or 54, 18, 6, 2, $\frac{2}{3}$, &c., decreasing by a uniform divisor, 3.

The multiplier, or divisor, which produces the series, is called the *ratio*.

1. A man bought 5 yards of cloth, paying 3 cents for the first yard, 6 for the second, and so on, doubling the price to the last. What was the price of the last yard?

$3\times2\times2\times2\times2=48$, the cost of the last yard.

From the above operation, it will be seen that the cost of the second yard is the product of the ratio multiplied by the cost of the first yard; and that the cost of the third yard, is the product of the second power of the ratio multiplied by the cost of the first yard, or the first term; and finally, that the cost of the fifth yard, or the last term, is the product of the fourth power of the ratio, multiplied by the cost of the first yard. It appears, also, that any term in the series, may be found by involving the ratio to a power less 1 than the number of terms, and multiplying that power by the first term.

NOTE. The process of involving the ratio to a high power, may be shortened by multiplying together those lower powers whose indices added equal the index of the power sought. To find the fifth power of 3, we may multiply together the second and third powers, for the index of the second power of 3, and the index of the third power added, equal 5, $3^{2+3}=3^{5}$, and $3^{2}\times3^{3}=9\times27=243$, the 5th power of 3.

When the first term and ratio are given, to find the last term.

RULE.

Involve the ratio to a power whose index is 1 less than the number of terms, and multiply this power by the first term: the product will be the answer, if the series is increasing; but if it is decreasing, divide the first term by the ratio.

QUESTIONS. 1. What is Geometrical Progression? 2. What is an ascending series? 3. What a descending? 4. What is the ratio? 5. When the first term and ratio are given, how do you find the last term?

2. If I hire a man for 12 months, and agree to pay him 1 dollar for the first month, 3 for the second, and so in a triple proportion, what must I pay him for the last month?

1 2 3 4 5+6=11
3 9 27 81 243×729=177147×1=$177147 *Answer.*

It will be seen, that the sum of the indices of the fifth and sixth powers added, equal 11, which 1 less than the number of terms; and the fifth and sixth powers multiplied together equal the 11th power of the ratio, which, multiplied by the first term, gives the answer, or the last term.

3. A man bought 20 cows, paying 2 farthings for the first, 10 for the second, and so on, in a five fold ratio. What was the price of the last cow?

Answer, £39736429850 5s. 2d. 2qrs.

4. A man bought 5 yards of cloth, giving 2 cents for the first, and 32 for the last; the prices forming a geometrical series, the ratio which was 2. What was the whole cost of the cloth?

The price of the cloth would be the sum of the following numbers—2+4+8+16+32=62, the whole cost. It will be seen, that the whole cost is the same as the difference between the two extremes divided by the ratio less 1 added to the greater extreme—Thus, 32—2=30 and 30÷1=30, and 30+32=62.

When the first term, last term, and ratio are given, to find the sum of the series, we have the following

RULE.

Divide the difference between the two extremes by the ratio less 1, and add the quotient to the greater term; their sums will be the answer.

2. The extremes of a Geometrical Progression are 3 and 18673, and the ratio 11; what is the sum of the series?

Operation.

$$\frac{18673-3}{11-1}+18673=20540, \text{ Answer.}$$

3. If I discharge a debt, by paying 1 dollar the first month, 4 the second, and so on, in a four fold ratio; the last payment was 65536 dollars; what was the whole debt?

Answer, $87381.

QUESTIONS. 6. When the first and last terms, and the ratio are given, how do you find the sum of the series?

4. The first term in a geometrical series is 2, the number of terms 10, and the ratio 3; what is the sum of the series? *Answer*, 59048.

Note. The last term may be found by Rule first, or the two processes of finding the last term and the sum of the series may be reduced to one—Thus:

When the first term, the number of terms, and the ratio are given, to find the sum of the series,

Involve the ratio to a power whose index is equal to the number of terms, from which subtract 1, divide the remainder by the ratio less 1, and the quotient multiplied by the first term will be the answer.

5. A man sold 15 yards of cloth; the first yard for 1 shilling, the second for 2, the third for 4, and so on, doubling the price of each succeeding yard; for how much did he sell the whole?

Operation.

$$2^{15}=32768, \text{ and } \frac{32768-1}{2-1}\times 1=32767\text{s.}$$

2|0)3276|7

£1638 7s. *Answer.*

6. A man bought 20 yards of cloth, agreeing to pay 3 pence for the first yard, 9 pence for the second, and so on, in a triple proportion to the last; what did he pay for the whole? *Answer*, £21792402 10s.

7. A gentleman bought a horse, agreeing to pay what his shoes would amount to, at 1 cent for the first nail, 2 for the second, 4 for the third, and so on, doubling the price of each succeeding nail to the last. The number of nails was 32; what was the price of the horse? *Answer*, $42949672,95.

8. A laborer wrought 20 days, and received for the first day's labor 4 grains of rye, for the second 12, for the third 36, &c. How much did his wages amount to, allowing 7680 grains to make a pint, and the whole to be disposed of at $1 per bushel? *Answer*, $14187.

COMPOUND INTEREST BY PROGRESSION.

1. What is the amount of \$6 for four years, at 6 per cent., compound interest?

Operation.

$$\begin{array}{rl}
\$6 & \text{1st term.}\\
1,06 & \\
\hline
636 & \text{2d.}\\
1,06 & \\
\hline
3816 & \\
636 & \\
\hline
6,7416 & \text{3d.}\\
1,06 & \\
\hline
404496 & \\
67416 & \\
\hline
7,146096 & \text{4th.}\\
1,06 & \\
\hline
42876576 & \\
7146096 & \\
\hline
7,57486176 & \text{5th.}
\end{array}$$

It will be seen, that this question may be solved by the RULE after Example 1st in Progression. The principal is the first term, the amount of \$1 for one year the ratio, and the number of years, 1 less than the number of terms. The question may be thus stated :—If the first term be 6, the number of terms 5, and the ratio 1,06, what is the last term?

$$1,06^{4}=1,262\times6=7,572 \text{ dollars.}$$

The amount of £1, or \$1, at 5 or 6 per cent., may be found by the table for Compound Interest, (see p. 166.)

2. What is the amount of \$30 for 7 years, at 5 per cent. compound interest? *Answer*, \$42,213.

3. What is the amount of \$7 for 4 years, at 9 per cent. compound interest? *Answer*, \$9,881.

4. If the amount of a certain sum for 6 years, at 6 per cent., compound interest, be $56,74040, what is that sum, or principal?

It will be seen, that this question is the reverse of the preceding. If the amount be the product of that power of the ratio denoted by the number of years and the principal, then the amount divided by that power of the ratio will be the principal.

$$\frac{56{,}74040}{1{,}06^{6}}=\$40\ \textit{Answer.}$$

5. If the amount of $40 for 6 years, compound interest, be $56,74040, what is the rate per cent.?

$$\frac{56{,}74040}{40}=1{,}41851=$$ the 6th power of the ratio; then by extracting the 6th root we have 1,06 for the ratio.

Answer, 6 per cent.

6. If the amount of $40, at 6 per cent., compound interest, be $56,74040, what is the time?

$$\frac{56{,}74040}{40}=1{,}51851=1{,}06$$ raised to a power whose index is equal to the time; therefore, if we divide 1,41851 by 1,06 until there is no remainder, it is plain, that the number of divisions will be the time required: or, having found the power of the ratio, we may look in the table under the given rate per cent., and against the power we shall find the number of years. *Answer*, 6 years.

7. In what time will $60 amount to $75,74820, at 6 per cent., compound interest? *Answer*, 4 years.

ANNUITIES AT COMPOUND INTEREST.

An annuity is a sum of money payable yearly for a certain number of years, or forever.

When annuities are not paid at the time they become due, they are said to be in arrears.

The sum of all the annuities remaining unpaid, together

Questions. 1. What is an annuity? 2. When are annuities said to be in arrears? 3. What is the *amount*?

with the interest on each, for the time they have remained due, is called the *amount*.

EXAMPLES.

1. What is the amount of an annual pension of $100, which has remained unpaid 5 years, allowing 6 per cent., compound interest?

The last year's pension will be $100, without interest, because it is paid as soon as due; the last but one will be $106, the amount of $100 for one year; the last but two, $112,36, the amount of $100 for two years at compound interest, and so on, forming a geometrical progression. The sum of these amounts will be the sum of the series, or the amount due. Hence, *when the annuity, time and rate per cent. are given, to find the amount, we have the following*

RULE.

Involve the ratio to a power denoted by the number of years; from this power subtract 1, divide the remainder by the ratio less 1, and the quotient, multiplied by the annuity, will be the amount.

The above example may be stated thus: If the first term be 100, the number of terms 5, and the ratio 1,06, what is the sum of all the terms?

$$\frac{1,06^{5}-1}{1,06-1}=100=563,7$$ *Answer*, $563,7.

2. What is the amount of an annuity of $70, to continue 5 years, allowing 6 per cent., compound interest?
Answer, $394,59.

3. What is the amount of an annuity of $160, to continue 10 years, at 5 per cent., compound interest?
Answer, $2012,448.

4. If a yearly rent of $75 be in arrears 4 years, to what does it amount, at 6 per cent., compound interest?
Answer, $328,087.

5. A salary of $600 remains unpaid 5 years. To what does it amount, allowing 6 per cent., compound interest?
Answer, $3382,255.

QUESTIONS. 4. What is the rule, when the annuity, time and rate per cent. are given, to find the amount?

The annuity, time, and rate given, to find the present worth.

Find the amount of the annuity in arrears for the whole time; this amount divided by that power of the ratio denoted by the number of years, will give the present worth.

6. What is the present worth of an annual pension of $96, to continue 4 years, allowing 6 per cent., compound interest? *Answer*, $332,643.

7. What is the present worth of an annual salary of $400, to continue 5 years, allowing 5 per cent., compound interest? *Answer*, $1731,792.

◆

TABLE,

Showing the present worth of $1 or £1 annuity, at 5 and 6 per cent., compound interest, for any number of years from 1 to 40.

years.	5 *per cent.*	6 *per cent.*	*years.*	5 *per cent.*	6 *per cent*
1	0,95238	0,94339	21	12,82115	11,76407
2	1,85941	1,83339	22	13,163	12,04158
3	2,72325	2,67301	23	13,48807	12,30338
4	3,54595	3,4651	24	13,79864	12,55035
5	4,32948	4,21236	25	14,09394	12,78335
6	5,07569	4,91732	26	14,37518	13,00316
7	5,78637	5,58238	27	14,64303	13,21053
8	6,46321	6,20979	28	14,89813	13,40616
9	7,10782	6,80169	29	15,14107	13,59072
10	7,72173	7,36008	30	15,37245	13,76483
11	8,30641	7,88687	31	15,59281	13,92908
12	8,86325	8,38384	32	15,80268	14,08398
13	9,39357	8,85268	33	16,00255	14,22917
14	9,89864	9,29498	34	16,1929	14,36613
15	10,37966	9,71225	35	16,37419	14,49825
16	10,83777	10,10589	36	16,54685	14,62098
17	11,27407	10,47726	37	16,71129	14,73678
18	11,68958	10,8276	38	16,36789	14,84602
19	12,08532	11,15811	39	17,01704	14,94907
20	12,46221	11,46992	40	17,15909	14,92640

NOTE. To find the present worth of any annuity, at 5 or 6 per cent., by the above table. First, find the present worth of $1 or £1 annuity; then, multiply it by the given annuity, and the product will be the present worth.

QUESTIONS. 5. Rule, when the amount, time, and rate are given, to find the present worth?

8. What is the present worth of an annuity of $300, to continue 25 years, at 6 per cent., compound interest?

The present worth of $1 annuity, by the table, for 25 years, is 12,78335; then, 12,78335×300=$3835,005, *Answer.*

9. What ready money will purchase an annuity of $250, to continue 40 years, at 6 per cent., compound interest?

Answer, $3731,6.

Annuities taken in reversion at compound interest.

Annuities taken in reversion are certain sums of money payable yearly for a limited period, but not to commence until after the expiration of a certain time.

RULE.

Find the present worth, to commence immediately, and this sum divided by the power of the ratio denoted by the time in reversion, will give the answer.

10. What is the present worth of a reversion of a lease of $40 per annum, to continue 6 years, but not to commence until the end of three years, allowing 6 per cent. to the purchaser?

$$\frac{\text{Present worth, } 198,9280}{\text{Third power of the ratio, } 1,19101}=165,147.$$

Answer, $165,147.

The same result may be obtained by finding the present worth of the annuity to commence immediately, and to continue the whole time. Thus 3+6=9 years, and from this sum subtract the present worth of the annuity for the time of reversion, 3 years. Or, by the table, find the present worth of $1 for the whole time; from the sum subtract the present worth of $1 for the time of reversion, and multiply the difference by the given annuity. Thus:

The whole time, 6,80169
The time of reversion, 2,67301

Difference, 4,12868
40

$165,14720 *Answer.*

QUESTIONS. 6. What are annuities taken in reversion? 7. Rule?

11. What is the present worth of \$50, payable yearly for 4 years, but not to commence until 2 years, at 6 per cent.? *Answer*, \$154,1965.

12. What is the present worth of the reversion of a lease of \$70 per annum, to continue 20 years, but not to commence until the end of 8 years, allowing 6 per cent. to the purchaser? *Answer*, \$503,7459.

13. What is the present worth of a lease of \$200, to continue 30 years, but not to commence until the end of 10 years, allowing 6 per cent.? *Answer*, \$1513,264.

To find the present worth of an annuity to continue forever.

RULE.

Divide the annuity by the rate per cent., and the quotient will be the present worth.

14. What is the present worth of a freehold estate whose yearly rent is \$60, allowing 6 per cent. to the purchaser?

$$\frac{60}{,06}=\$1000, \textit{ Answer.}$$

It is evident, that the estate is worth as much money, as, at the given rate per cent., would give interest equal to the rent.

15. What is \$300 annuity worth, to continue forever, allowing 5 per cent. to the purchaser? *Ans.*, \$6000.

To find the present worth of a freehold estate, in reversion at compound interest.

RULE.

Find the value, as though it were to be entered on immediately, by the foregoing rule, and divide this value by that power of the ratio denoted by the time of reversion, and the quotient will be the present worth of the estate in reversion.

16. Suppose a freehold estate, of \$48 per annum, to commence two years hence, be put on sale. What is the value, allowing 6 per cent. to the purchaser?

$$\frac{48}{,06}=800 \quad \text{then} \quad \frac{800}{1,06^2=1,1236}=\$711,997 \textit{ Answer.}$$

QUESTIONS. 8. Rule for finding the present worth of an annuity to continue forever? 9. Rule for finding the present worth of a freehold estate in reversion at compound interest?

17. Which is the more valuable, a term of 16 years, in an estate of $100 per annum, or the reversion of such an estate forever after 16 years, computing at the rate of 5 per cent. compound interest?

Answer, The term of 16 years, by $167,565.

PERMUTATION.

Permutation is the method of finding how many changes may be made upon the order of any given number of things?

1. How many changes can be made of the first three letters of the alphabet?

The letter *a* can occupy but one position; *a* and *b* can change places, and occupy 2 positions, *ab* and *ba*; 1×2=2. The three letters, *a*, *b* and *c*, can, any two of them, leaving out the third, have two positions 1×2=2, consequently, when the third is taken in, there can be 1×2×3=6 positions which may be thus expressed. Thus, *abc*, *acb*, *bac*, *bca*, *cba*, *cab*. The same may be shown of any number of things. Hence, to find the number of changes which can be made of any given number of different things:

RULE.

Multiply all the terms of the natural series of numbers, from 1 up to the given number, continually together, and the last product will be the answer required.

2. Christ Church, in Boston, has 8 bells. How many changes may be rung upon them?

1×2×3×4×5×6×7×8=40320, *Answer*.

3. Six men met at a public house, and agreed to remain so long as they could occupy different situations at the dinner table. How long did they remain, and what was the price of their board, at 25 cents for each dinner?

Answer, { 720 days.
$1080 board.

Questions. 1. What is Permutation of quantities? 2. Rule for finding the number of permutations?

POSITION.

Position teaches to find the true number by the use of false, or supposed, numbers.

It is of two kinds—Single and Double.

Single Position is so called, because the true number is obtained by the use of *one* supposed number.

1. A, B and C travelled. C paid a certain part of the expense; B paid double, and A treble the sum which C paid. The amount of their expenses was $60. What did each one pay?

Suppose C's expense was $8; then by the conditions of the question, B's expense $8\times2=\$16$; and A's $8\times3=\$24$; and the sum of their expenses $\$8+\$16+\$24=\$48$. As the ratios, in the true and supposed, are the same, it follows, that the true sum of their expenses will have the same ratio to the true expense of each individual that the sum of their supposed expenses has to the supposed expenses of each individual. Thus:

48 : 8 : : 60 : 10 C's expense; and
48 : 16 : : 60 : 20 B's expense; and
48 : 24 : : 60 : 30 A's expense.

RULE.

Suppose any number, and proceed in the operation as though it were the true; then, as the result of the operation, or sum of the errors, is to the supposed number, so is the given number to the true number required.

EXAMPLES.

2. A person, after spending $\frac{1}{2}$ and $\frac{1}{3}$ of his income, had $30 left. What had he at first?

Suppose $60
$\frac{1}{2}=30$
$\frac{1}{3}=20$

60—50=10 income left:
Then 10 : 60 : : 30 : 180 *Answer.*

Or by fractions: $\frac{1}{2}=\frac{3}{6}$, and $\frac{1}{3}=\frac{2}{6}$; then $\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$, the income spent, and $\frac{1}{6}$ remains=$30; then $\frac{6}{6}=30\times6=\$180$, as before.

Questions. 1. What is Position? 2. What is Single Position? 3. Rule?

3. A certain sum of money is to be divided between 5 men, in such a manner that A shall have $\frac{1}{4}$, B $\frac{1}{5}$, C $\frac{1}{10}$, D $\frac{1}{20}$, and E the remainder, which is $40. What is the sum?
Answer, $100.

4. A schoolmaster being asked how many scholars he had, replied, if he had as many more $\frac{1}{2}$, and $\frac{1}{4}$ as many more, he would have 11 less than 99. How many had he?
Answer, 32.

5. A man bought a horse, chaise and harness for $216. The horse cost twice as much as the harness, and the harness one third as much as the chaise. What was the cost of the chaise? *Answer*, $108.

6. What number is that whose $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{10}$, $\frac{1}{9}$, $\frac{1}{5}$ and $\frac{1}{6}$ make 127? *Answer*, 90.

7. A man being asked his age, said, If you add to its double $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{9}$ and $\frac{1}{15}$ of my age, it will be 122. What was his age? *Answer*, 45.

8. A certain sum of money is to be divided among 4 persons, in such a manner that the first shall have $\frac{3}{4}$ of $\frac{1}{2}$ of $\frac{2}{3}$; the second $\frac{1}{11}$ of $\frac{11}{12}$ of 2; the third, $\frac{1}{45}$ of $\frac{45}{8}$; the fourth has $110. What is the sum divided?
Answer, $240.

9. A and B having found a purse of money, disputed who should have it. A said that $\frac{1}{5}$, $\frac{1}{10}$ and $\frac{1}{20}$ of it amounted to $35, and if B would tell him how much was in it, he should have the whole, otherwise he should have nothing. How much did the purse contain?
Answer, $100.

DOUBLE POSITION.

Double Position teaches to discover the true, by the use of *two* supposed, numbers.

RULE.

I. Suppose two numbers, and proceed with each according to the conditions of the question, as in Single Position, noting the error. The difference between the result and the given sum is the error.

Questions. 4. What is Double Position? 5. On what supposition is this Rule founded? 6. Rule for the operation?

II. Multiply the first supposition by the second error, and the second supposition by the first error.

III. If the errors are alike—that is, both too great or both too small, divide the difference of the products by the difference of the errors.

IV. If the errors are unlike—that is, one too large, and the other too small, divide the sum of the products by the sum of the errors.

EXAMPLES.

1. A man being asked what his carriage cost, replied, If it had cost twice as much as it did, and $20 more, it would have cost him $370· What was the cost of the carriage?

Suppose first $120
2
240
20
260

Having supposed 120, and proceeded with it according to conditions of the question, the result obtained is 260; then 370—260=110, the first error.

Suppose, secondly, $160
2
320
20
340

Then 370—340=30, the second error.

First sup. 120 110 first error.
X
Second sup. 160 30 second error.
110 120
17600 3600
3600

110—30=80)14000(175 *Answer.*
80
600
560
400
400

Note. This rule is founded on the supposition that the first error is to the second as the difference between the true and first supposed number is to the difference between the true and second supposed number. When this is not the case, the exact number cannot be obtained by this rule.

2. A, B and C built a house, which cost $228; B paid $30 more than A, and C paid as much as A and B; what did each pay?

Answer, A paid $42,
B paid $72,
C paid $114.

3. A and B have the same income. A saves $\frac{1}{4}$ of his annually, but B, by spending $120 per annum more than A, at the end of 6 years finds himself $120 in debt; what is their income, and what does each spend annually?

Answer, Their income $400.
A spends $300, and
B spends $420.

4. A man has two silver cups of unequal weight, having one cover to both, weighing 5 oz.; when the cover is put on the less cup, it weighs double the greater; when put upon the greater cup, it weighs three times the less; what is the weight of each cup?

Ans., The less, 3 oz.
The greater, 4oz.

5. There is a fish whose head is 3 feet long, his tail is as long as his head and half the length of his body, and his body is as long as his head and tail; what is the length of the fish? *Answer,* 24 feet.

6. A man being asked, in the afternoon, what o'clock it was, answered, that the time past from noon, was equal to $\frac{1}{8}$ of the time from midnight; required the time?

Answer, 30 minutes past 1 o'clock.

7. A gentleman has two horses, and one carriage which is worth $100. If the first horse be harnessed into the carriage, he and the carriage together will be worth three times as much as the second horse; but if the second be harnessed into the carriage, they will be worth seven times as much as the first horse; what is the value of each horse?

Answer, $20 and $40.

8. A laborer was hired 60 days upon this condition, that, for every day he wrought he should receive 3s. 4d., and for every day he was idle he should forfeit 1s. 8d. At the expiration of the time, he received £3 15s. How many days did he work, and how many days was he idle?

Answer, He was employed 35 days,
and was idle 25.

ALLIGATION.

Alligation is the method of mixing two or more simples, of different qualities, so that the composition may be of a mean, or middle quality.

When the quantities and prices of the simples are given, to find the mean price of the mixture compounded of them, the process is called

ALLIGATION MEDIAL.

1. If I mix 8 lbs. of sugar, worth 10 cents a pound, with 10 lbs., worth 15 cents a pound, what is 1 pound of the mixture worth?

Eight pounds at 10 cents a pound are worth $10\times8=80$ cents, and 10 lbs. at 15 cents are worth $15\times10=150$ cts.; then $80+150=230$ cts., the price of the whole mixture, and $8+10=18$ lbs., the whole mixture; then \$2,30÷18 lbs.= $12\frac{7}{9}$ cts., the worth of 1 lb. of the mixture. Hence the

RULE.

Multiply each quantity by its price, and divide the sum of the products by the sum of the quantities; the quotient will be the rate of the compound required.

EXAMPLES.

2. A grocer mixes sugar, 5 lbs. at 6 cents, 8 lbs. at 5 cts., and 7 lbs. at 10 cents a pound; what is 1 pound of the mixture worth? *Answer*, 7 cents.

3. A farmer mixes 12 bushels of wheat, at \$1,75 a bushel, 8 bushels of rye at \$1, and 6 bushels of corn at 80 cts. a bushel; what is a bushel of the mixture worth? *Answer*, \$1,30.

4. A goldsmith melted together 12 lbs. of gold, 21 carats fine, 8 lbs. 20 carats fine, 9 lbs. 22 carats fine, and 7 lbs. 18 carats fine; of what fineness is the mixture? *Answer*, $20\frac{4}{9}$ carats fine.

5. A merchant mixed 8 gallons of wine, at 4s. 2d. per gallon, with 10 gallons at 6s. 5d., and 12 gallons at 8s. 4d. per gallon; what is a gallon of the mixture worth? *Answer*, 6s. 7d.

6. If 4 lbs. of tea at 6s. per lb., 8 lbs. at 5s., and 6 lbs. at 3s. be mixed together, what is 1 lb. of the mixture worth? *Answer*, $4\frac{5}{9}$ shillings.

ALLIGATION ALTERNATE.

Alligation Alternate is, when the prices of the simples to be mixed, and the mean rate are given, to find what quantity of each is to be taken at a given rate.

1. I have corn at 50 cents a bushel, and oats at 30 cents a bushel, which I would mix, so that the mixture may be worth 40 cents a bushel; what quantity of each must be taken?

It is evident that equal quantities of each must be taken, for the price of the corn exceeds the mean as much as the price of the oats falls short of it, which is 10 in each case. We find, also, that the whole mixture, which is 20 bushels, at the mean rate, 40 cents a bushel, equals the price of 10 bushels of oats at 30, and 10 bushels of corn at 50 cents a bushel.

RULE.

I. Reduce the rates of all the simples to the same denomination, and write them in a column under each other, and the mean rate on the left hand.

II. Connect the rate of each simple, which is less than the rate of the compound, with one that is greater, and each that is greater with one that is less.

III. Write the difference between each rate, and that of the compound against the number with which it is connected. Then, if only one difference stand against any rate, it will express the relative quantity to be taken of that rate, but if more than one, their sum will express that quantity.

EXAMPLES.

2. A farmer has wheat at \$1,50, rye at \$1,00, corn at 90, and oats at 40 cents a bushel, which he mixes so that the mixture is worth 95 cents a bushel; what quantity of each does he take?

Operation.

$$\begin{array}{ll} & \text{bushels.} \\ 95\left\{\begin{array}{l} 1,50 \\ 1,00 \\ ,90 \\ ,40 \end{array}\right. & \begin{array}{r} 55 \\ 5 \\ 5 \\ 55 \end{array}\left.\vphantom{\begin{array}{l}1\\1\\1\\1\end{array}}\right\}\textit{Ans.} \end{array} \qquad \text{Or} \qquad \begin{array}{ll} & \text{bushels.} \\ 95\left\{\begin{array}{l} 1,50 \\ 1,00 \\ ,90 \\ ,40 \end{array}\right. & \begin{array}{r} 5 \\ 55 \\ 55 \\ 5 \end{array}\left.\vphantom{\begin{array}{l}1\\1\\1\\1\end{array}}\right\}\textit{Ans.} \end{array}$$

By linking the price of the different simples as above, their quantities are mutually mixed, and the portion taken of each depends upon the manner of linking them. In the first operation, the price of the wheat, which is greater than the mean price, is linked with the price of the oats, which is less. The price of the wheat is found to be as much greater than the mean rate, as the price of the oats is less; therefore an equal quantity of each is taken. The same is true of the corn and oats. In the second operation, the price of the wheat is linked with the price of the corn. The difference between the price of the wheat and the mean rate, is 55, and the difference between the price of the corn and the mean rate is 5. Hence, it appears that the less the difference between the price of a simple and the mean rate, the greater will be the quantity taken of that simple, and the greater the difference the less the quantity.

3. A merchant has teas at 72 cents, at 62 cents, and 57 cents a pound, which he would mix, so that the mixture may be worth 67 cents per lb.; what quantity of each must be taken?

Operation. lbs.

$$67\left\{\begin{array}{l|l}72 & 10+5=15\\ 62 & 5=\quad 5\\ 57 & 5=\quad 5\end{array}\right\}\textit{Answer.}$$

The correctness of the above operation may be ascertained thus: The cost of 15 lbs. at 72 cents, is $10,80, and the cost of 5 lbs. at 62 cents, is $3,10, and the cost of 5 lbs. at 57 cents, is $2,85. Then the whole cost is $10,80 +$3,10+2,85=$16,75, which divided by 25 lbs., $16,75 ÷25=67 cents, the mean price of the mixture. Hence, it appears that Alligation Alternate is the reverse of Alligation Medial, and may be proved by it.

4. A grocer mixes wines at 29s., 24s., 22s., and 17s. a gallon, so that the mixture is worth 23s. per gallon. How much of each sort does he take?

1st *Answer.*
1 gal. at 29s.
6 gal. at 24s.
6 gal. at 22s.
1 gal. at 17s.

2d *Ans.*
6 gal. at 29s.
1 ‘ at 24s.
1 ‘ at 22s.
6 ‘ at 17s.

3d *Answer.*
7 gal. at 29s.
6 ‘ at 24s.
6 ‘ at 22s.
7 ‘ at 17s.

As many different answers may be obtained to questions in this rule as there are modes of linking the prices of the simples. Let the number of simples be what it may, and with how many soever each one is linked, since the price of one that is less than the mean rate is always linked with one that is greater, there will always be an equal balance of loss and gain between the two, and consequently an equal balance on the whole.

5. It is required to mix brandy at 12s.; wine at 9s.; cider at 2s.; beer at 1s., and water at 0s. per gallon, so that the mixture may be worth 7s. per gallon; what quantity of each must be taken?

$$\textit{Answer,}\left\{\begin{array}{l}\text{13 gal. of brandy.}\\ \text{5 - - - - - wine.}\\ \text{2 - - - - cider.}\\ \text{5 - - - - - beer.}\\ \text{5 - - - - water.}\end{array}\right.$$

When the composition is limited in quantity.

RULE.

Find the proportion of each quantity as before; then say, as the sum of the quantities : is to the given quantity : : so is each of the differences : to the required quantity.

EXAMPLES.

6. Suppose a mass of pure gold, a mass of pure silver, and a mass which is a mixture of gold and silver, each weighing 9 ozs.; by immersing them in water, it is found that the quantity of water displaced by the gold is 5; by the silver 8, and by the mixture 7. What part of the mixture is gold, and what part silver?

$$7\left\{\begin{array}{l}5\text{—}\\ 8\text{—}\end{array}\right.\left|\begin{array}{l}1\\ 2\end{array}\right.$$

$$\overline{3}:9::\left\{\begin{array}{l}1:3\text{ gold.}\\ 2:6\text{ silver.}\end{array}\right.$$

By a similar problem, Archimedes detected the fraud of the artist employed by Hiero, king of Syracuse, to make him a crown of pure gold.

7. A druggist has medicines at 6d., 3d., 9d., and 4d. per oz., and would form a compound of 15 ozs., worth 5d. per oz.; how much of each sort must he take?

$$\textit{Answer,}\left\{\begin{array}{l}1\frac{7}{8}\text{ oz. at 6d.}\\ 7\frac{1}{2}\text{ - - - 3d.}\\ 3\frac{3}{4}\text{ - - - 9d.}\\ 1\frac{7}{8}\text{ - - - 4d.}\end{array}\right.$$

8. A goldsmith would melt together gold of 13, of 14, of 15, and of 21 carats fine, to form a composition of 35 ozs. 18 carats fine. What proportion of each must he take?

$$\textit{Answer,}\left\{\begin{array}{rcl}5 & \text{of} & 13\\ 5 & \text{``} & 14\\ 5 & \text{``} & 15\\ 20 & \text{``} & 21\end{array}\right\}\text{car. fine.}$$

9. How many gallons of water worth 0s. per gal., must be mixed with wine worth 12s. per gallon, so as to fill a cask of 20 gallons, and that a gallon of the mixture may be afforded at 9s. per gallon?

$$\textit{Answer,}\left\{\begin{array}{l}5 \text{ gal. water.}\\ 15 \text{ gal. wine.}\end{array}\right.$$

When one of the simples is limited to a certain quantity.

RULE.

Find the proportional quantities, or differences, as before; then say, As the difference standing against the given quantity : is to the given quantity, : : so are the other differences severally : to the several quantities required.

EXAMPLES.

10. A grocer mixes sugar at 9 cts., 12 cts., and 14 cts., with 16 lbs. at 15 cts.; how much of each sort must he take, that the mixture may be worth 13 cts. per lb.?

$$13\left\{\begin{array}{l|l}9 & 2\\ 12 & 1\\ 14 & 1\\ 15 & 4 \text{ against the given quantity.}\end{array}\right.$$

$$4 : 16 : : \left\{\begin{array}{lll}2 : 8 \text{ lbs. at} & 9\text{s.}\\ 1 : 4 \quad \text{`` ``} & 12\text{s.}\\ 1 : 4 \quad \text{`` ``} & 14\text{s.}\end{array}\right\}\textit{Answer.}$$

11. A grocer would mix flour at \$6, \$5, \$12 a barrel, with 10 barrels at \$11 a barrel; how much of each kind must he take, that the mixture may be worth \$11 a barrel?

$$\textit{Answer,}\left\{\begin{array}{lcl}4 & \text{at} & \$6\\ 2 & \text{``} & \$5\\ 8 & \text{``} & \$12\end{array}\right\}\text{per barrel.}$$

12. How much water must be mixed with 100 gallons of brandy, worth 7s. 6d. per gallon, to reduce it to 6s. 3d. per gallon? *Answer*, 20 gallons.

13. A farmer would mix barley at 50 cents, oats at 30, with 20 bushels of rye at 60 cents a bushel; how much of each sort must he take, that the provender may be worth 40 cents per bushel?

Answer, { 60 bushels of oats and
20 bushels of barley.

DUODECIMALS.

This rule is principally used in measuring surfaces and solids. Calculations are generally made in feet, inches, or primes, seconds, thirds, fourths, and so on. These subdivisions of the foot are made by a common divisor, 12, an inch being $\frac{1}{12}$ of a foot, a second $\frac{1}{12}$ of an inch, or $\frac{1}{144}$ of a foot, thus forming a descending series of a geometrical progression, whose first term is 1, and the ratio 12. Hence the term *duodecimal.* It is derived from the Latin word *duodecim,* which signifies twelve. Duodecimals, then, are fractions of a foot, as may be seen by the following

TABLE I.

1′ inch, or prime, is - - - - -	$\frac{1}{12}$	of a foot.
1″ second is $\frac{1}{12}$ of $\frac{1}{12}$ - - - - -	$\frac{1}{144}$	" "
1‴ third is $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ - - -	$\frac{1}{1728}$	" "
1⁗ fourth is $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$	$\frac{1}{20736}$	" "

The marks ′, ″, ‴, ⁗, placed over numbers, denote the denomination, and are called *indices.* In Multiplication of Duodecimals, the denomination of the product is denoted by the sum of the indices. That is, if inches be multiplied by inches, the product will be seconds—thus, 2′×2′=4″. If inches be multiplied by seconds, the product will be thirds—thus, 2′×2″=4‴, &c.

TABLE II.

12⁗	fourths make - - - - - - - -	1‴ third,
12‴	thirds - - - - - - - - - -	1″ second,
12″	seconds - - - - - - - -	1′ inch, or prime,
12′	inches, or primes, - - - - -	1 foot.

Questions. 1. What are Duodecimals? 2. For what is the rule chiefly used? 3. What are the divisions of the foot?

MULTIPLICATION OF DUODECIMALS.

1. How many square feet in a board, 11 feet 2 inches long, and 1 foot 4 inches wide?

Having written numbers of the same denomination under each other, as in Multiplication of Compound Numbers, (see p. 71) we commence with the feet in the multiplier, and say, once $\frac{2}{12}$ is $\frac{2}{12}$, and 11 feet multiplied by 1, is 11. Proceeding to the next figure in the multiplier, which is 4 inches, or $\frac{4}{12}$, we say, 4 times 2 are 8, but 4 is $\frac{4}{12}$, and 2 is $\frac{2}{12}$; therefore $\frac{4}{12}\times\frac{2}{12}=\frac{8}{144}$ of a foot, or 8″ seconds, which being less than 1 prime, we write it in the place of seconds, and proceed to the next figure in the multiplicand, which is 11. Multiplying 11 feet by $\frac{4}{12}$, we have $11\times\frac{4}{12}=\frac{44}{12}=3$ feet 8′ inches. Having written 8 in the place of inches, and 3 in the place of feet, we add the several partial products, and obtain 14 feet 10′ 8″, the answer. By examining the foregoing operation, it will be seen, that the first product, being the product of inches by feet, is inches. The second product is the product of feet, and consequently is feet. The third product is the product of inches by inches; the sum of the indices being two, it is 12ths of an inch. The fourth and last is the product of feet by inches, and is 12ths of a foot. Therefore, to multiply feet, inches, &c., by numbers of corresponding denominations, we have the following

Operation.

$$\begin{array}{rrl} 11 & 2' & \\ 1 & 4' & \\ \hline 11 & 2' & \\ 3 & 4' & 8'' \\ \hline 14 & 10' & 8''\ \textit{Ans.} \end{array}$$

RULE.

I. Write the several denominations of the multiplier under the corresponding denominations of the multiplicand.

II. Multiply first the lowest denomination in the multiplicand by the highest in the multiplier, observing to carry 1 for every 12 from a lower to a higher denomination.

It is to be remembered, that the denomination of the product of two numbers is denoted by the sum of the indices.

EXAMPLES.

2. How many square feet in a marble slab, 5 feet 7 inches in length, and 4 feet 8 inches in breadth?

☞ *For Operation, see next page.*

15 – 8
14 – 9

Operation.

5 7′
4 8′

22 4′
3 8′ 8″

26ft. 0′ 8″ *Answer.*

3. How many square feet in a room, 15 feet 8 inches in length, and 14 feet 9 inches in breadth?
Answer, 156 ft. 1 in.

4. What is the product of 15 ft., 2′, 6″×20 ft. 3′, 7″?

5. How many solid feet in a block 4 ft., 8′ long, 3 ft., 6′ wide, and 2 ft., 9′ thick? *Answer*, 44 ft. 11′.

NOTE. The solid contents may be found by multiplying the length by the breadth, and that product by the thickness.

6. How much wood in a load 9 ft. 8′ long, 8 ft. 7′ wide, and 3 ft. high? *Answer*, 1 cord, 120 ft., 11′.

7. How many square feet in a stock of 20 boards, 13 ft. 11′ long, and 1 ft. 7′ wide? *Answer*, 440 ft. 8′ 4″.

8. How many feet of flooring in a room 30 feet 6 inches long, and 19 feet 5 inches in width?
Answer, 592 ft. 2′ 6″.

9. How much wood in a cubic pile, 12 feet 3 inches on each side, and what will it be worth at $4,25 per cord?
Answer to the last, $61,035.

10. How many square yards in the walls of a room, 14 feet 8 inches long, 11 feet 6 inches wide, and 7 feet 11 in. high? *Answer*, 46 yds. 0 ft. 0′ 10″ 8‴.

11. How many cord feet in a pile of wood 38 feet long, 7 feet 2′ wide, and 4 feet 7′ in height?
Answer, 78 cord feet, 2 in. 4″.

12. How many cord feet in a load of wood 8 feet long, 3 feet 6 inches high, and 4 feet 5 inches wide.
Answer, 7 cord feet; 11 solid feet 8 in.

13. How much wood in a load 9 feet long, 3 feet 4 in. wide, and 2 feet 6 inches high?
Answer, 4 cord feet; 11 solid feet.

NOTE. Many mechanics and surveyors take dimensions in feet and *decimal* parts. This method is preferable, inasmuch as by it the calculations of the artificer are rendered more simple and easy. For such, it is convenient to have a rule, or scale, four feet long, divided into feet, and each foot into ten equal parts. One foot, on one end of the rule, should be divided into one hundred equal parts. The former division will be 10ths, and the latter 100ths of a foot. Dimensions taken by this rule are calculated the same as other decimal fractions.

14. How many square feet in a board 20,5 in length, and 1,8 in width? *Answer*, $36\frac{9}{10}$ feet.

15. How much wood in a pile 40,5 in length, 5,4 in width, and 6,2 in height,—and what will it be worth at $3,75 a cord? *Answer, to the last*, $39,853.

16. What will be the cost of a marble slab, 13,9 feet in length, and 2,1 feet in width, at $1,18 per square foot? *Answer*, $34,444.

MISCELLANEOUS RULES.

MENSURATION.

I. *Mensuration of Surfaces.*

The area of a figure is the space contained between the bounding lines of its surface, without regard to thickness. The area is reckoned so many square inches, square feet, square yards, or square rods, &c.

To find the area of a Square, or Parallelogram.

Rule. Multiply the length by the breadth, or perpendicular height, and the product will be the area.

1. How many square rods in a field 28 rods on each side? $28 \times 28 = 784$ rods, *Answer*.

2. What is the area of a square field, one side of which is 25,35 chains? *Answer*, 642,622 chains.

3. What is the area of a field 30,5 chains in length, and 24,5 in width? *Answer*, 747,25 chains.

4. How many square feet in a board 18,8 feet long, and 2,7 feet wide? *Ans.*, 50,76 feet.

5. How many acres in a rectangular piece of ground, 64 rods long and 24 rods wide? *Ans.*, $9\frac{3}{5}$ acres.

To find the area of a triangle.

Rule. Multiply the perpendicular by the base, and one half the product will be the area; or multiply the base by half the perpendicular height, and the product will be the area.

---

1. What is the area of a triangle whose base is 20 feet, and whose height 18 feet?

$18\times20=360\div2=180$ feet, *Ans.*

2. What is the area of a triangle whose base is 55 rods, and its height 24,6 rods? *Ans.*, 676,5.

3. How many feet of boards will it take to cover the gable end of a barn 38 feet wide, the height from the beam to the top being 12,5 feet? *Ans.*, 237,5 feet.

When the three sides of a triangle are known, the area may be found by the following

RULE.

Add together the three sides, and from half their sum subtract each side separately; multiply the half sum and the remainders together continually, and the square root of the product will be the area.

4. What is the area of a triangle whose three sides are 14, 12 and 8 rods?

$14+12+8=34\div2=17$ the half sum: then,

$$\begin{array}{ccc}17 & 17 & 17\\ 14 & 12 & 8\\ \hline\end{array}$$

Rem. $3\times5\times9\times17=2295$: then $\sqrt{2295}=47,3+$ rds.

5. The three sides of a triangle are 6, 8 and 10 chains; what is the area? *Ans.*, 24 chains.

To find the area of a trapezoid.

Rule. Multiply half the sum of the two parallel sides by the perpendicular distance between them, and the product will be the area.

1. What is the area of a piece of land that is 30 chains long, 20 chains wide at one end, and 18 chains at the other? $20+18=38\div2=19$, the half sum of the two sides: then $19\times30=570$ chains, *Ans.*

2. What is the content of a board 10 feet long, 10 inches wide at one end, and 2 feet 10 inches at the other? *Ans.*, 18⅓ feet.

3. What is the area of a hall, 40 feet long, and at one end 30 feet, and at the other 24 feet wide? *Ans.*, 1080 feet.

4. How many acres in a farm 300 rods long, 80 rods wide at one end, and 60 at the other?
Answer, 131 acres, 40 rods.

To measure any irregular plane figure.

RULE.

The whole may be divided into triangles, and measured separately; the sum of the area of the triangles will be the area of the whole.

Of the Circle.

The circumference of a circle is found by calculation to be about $3\frac{1}{7}$ times the diameter, or more accurately by decimals, as 1 is to 3,1416, or as 113 is to 355, so is the diameter to the circumference. Hence, if the diameter is given, to find the circumference, it may be found by multiplying the diameter by $3\frac{1}{7}$, or by 3,11416, or as 113 is to 355, so is the diameter to the circumference.

1. What is the circumference of a circle whose diameter is 42 feet?

$42\times3\frac{1}{7}=132$ ft.; or $42\times3{,}1416=131{,}9472$ ft.;
or $113:355::42:131{,}946+$ feet.

By reversing the foregoing, the diameter may be found, the circumference being given.

2. If the circumference of a circle be 132 feet, what is the diameter? $132\div3\frac{1}{7}=42$ feet, *Answer*.

3. Suppose the diameter of a circular pond to be 121 rods, what is the circumference? *Ans.*, 380,28+rds.

4. If the circumference of a circular field be 198 rods, what is the diameter? *Answer*, 63 rods.

5. What is the diameter of a tree, whose circumference is $9\frac{3}{7}$ feet? *Answer*, 3 feet.

6. Suppose the circumference of the earth to be 25000, 8528 miles, what is the diameter? *Answer*, 7958 miles.

7. The diameter of the earth being 7958 miles, what is the circumference? *Answer*, 25000,8528.

To find the area of a circle.

RULE.

Multiply half the diameter by half the circumference; the product will be the area.

1. What is the area of a circular grove, whose diameter is 147 rods, and circumference 462 rods?

Answer, $462\div2\times147\div2=16978\frac{1}{2}$ rods.

2. What is the area of a circle whose diameter is 28, and the circumference 88 rods? *Answer*, 616 rods.

3. How many square rods in a circle whose circumference is 63, and the diameter 20 rods?

Answer, 315 rods.

The diameter given, to find the area.

Rule. Multiply the square of the diameter by ,7854, and the product will be the area.

1. What is the area of a circle whose diameter is 28 rods? $28\times28\times{,}7854=615{,}7536$ rods, *Answer*.

2. What is the area of a circle whose diameter is 59 rods? *Answer*, 2733,9774 rods.

The circumference given, to find the area.

Rule. Multiply the square of the circumference by ,07958, and the product will be the area.

1. What is the area of a circle whose circumference is 46 rods? $46\times46\times{,}07958=168{,}39128$ rods, *Answer*.

2. What is the area of a circle whose circumference is 44 rods? *Answer*, 154,06688 rods.

To find the area of an oval, or ellipsis.

Rule. Multiply the longest and shortest diameters together, and the product by ,7854; the last product will be the area.

1. What is the area of an oval, whose longest diameter is 7 feet, and the shortest 5 feet?

$7\times5\times{,}7854=27489$ feet, *Answer*.

2. What is the area of an oval, whose longest diameter is 15, and the shortest 13 feet? *Answer*, 153,153 ft.

To find the area of a globe or sphere.

Rule. Multiply the circumference by the diameter; the product will be the area.

1. What is the area of a globe, whose circumference is 44 feet, and the diameter 14? $44\times14=616$ ft. *Ans.*

2. How many square inches in the surface of a ball, 1 inch in diameter? *Answer*, 3,1416 inches.

3. How many square miles in the surface of the earth, allowing its circumference to be 25,000, and its diameter 8,000 miles. *Answer*, 200000000 miles.

MENSURATION OF SOLIDS.

The Mensuration of Solids includes the mensuration of all bodies which have length, breadth and thickness.

To find the solidity of a cube.

Rule. Multiply the length, breadth and thickness together, and the product will be the area.

1. If the length of one side of a cubical block be 14 inches, what is its solidity?

$14\times14\times14=2744$ inches, *Answer*.

2. How many cubical feet in a mound, each side of which is 25,5 feet? *Answer*, 16581,375 ft.

3. How many solid feet will be occupied by 21 chests, which are each 3 feet 3 inches every way?

Answer, 720,89+.

To find the solidity of a prism, or cylinder.

Rule. Find the area of the end, and multiply it by the length; the product will be the area.

1. What is the solidity of a prism, the area of whose end is 2,6 feet, and whose length is 16 feet?

$2{,}6\times16=41{,}6$ feet, *Answer*.

2. How many solid feet in a round log, 28 inches in diameter, and 15 feet in length? *Ans.*, 64,410 feet.

To find the side of the largest stick of timber that can be hewn from a round log.

Rule. Extract the square root of double the square of half the diameter at the smallest end of the stick, for the side of the stick when squared.

1. What will be the side of the largest stick of square timber which can be hewn from a round log, 18 inches in diameter at the smallest end?

$\sqrt{}9\times9\times2=12{,}727+$ in. *Answer*.

2. The diameter of a log at the smallest end is 24 inches; what will be the side of the largest stick of timber that can be hewn from it? *Answer*, 16,97+ inches.

To find the solidity of a pyramid, or cone.

RULE. Multiply the area of the base by $\frac{1}{3}$ of the height, and the product will be the area.

1. What is the content of a cone whose height is 21 ft., and the diameter of the base 9,5 feet?

$9{,}5\times9{,}5\times7854\times21\div3=496{,}176$ feet, *Answer.*

2. How many solid feet in a cone whose height is 48 feet, and whose diameter at the base is 13 feet?

Answer, 2123,7216 feet.

3. What is the solid content of a triangular pyramid, whose height is 72 feet, and the sides of its base 13,14 and 15 feet? *Answer*, 2184 feet.

To find the solid content of a globe, or sphere.

RULE. Multiply the cube of the diameter by ,5236, or multiply the square of the diameter by $\frac{1}{6}$ of the circumference.

1. What is the solidity of a ball, 9 inches in diameter?

$9\times9\times9\times{,}5236=381{,}7044$, *Answer.*

2. What is the solidity of a globe, whose diameter is 13 inches? *Answer*, 1150,3492 inches.

3. What is the solid content of the earth, its circumference being 25000 miles? *Ans.*, 26385814912 miles.

GUAGING.

GUAGING *is the art of measuring all kinds of vessels, such as pipes, hogsheads, barrels, &c.*

RULE.

Add the square of the head diameter to the square of the bung diameter; multiply the sum by the length, and the product by ,0014 for ale gallons, or by ,0017 for wine gallons.

1. What is the content of a cask, whose diameters are 18 and 26 inches, and its length 38 inches? $26\times26+18\times18\times38=38000$; then $38000\times{,}0017=64{,}6$ wine measure; $38000\times{,}0014=53{,}2$ gallons, beer measure.

2. How many wine gallons will fill a cask 50 inches in length, bung diameter 38, head diameter 30 inches?

Answer, 199,24 gal.

MEASURING GRAIN, &c.

When the grain is heaped in the form of a cone.

Rule. Measure the perpendicular height of the heap, and also the slanting height, from the top to the floor, in inches; then multiply the difference of the squares of those two heights by the perpendicular height, and this product by ,0005; the last product will be the content in bushels.

1. How many bushels in a parcel of wheat heaped in the form of a cone; the perpendicular height being 40 inches, and the slanting height 90 inches?

The square of 90=8100
The square of 40=1600

6500 difference of squares.
40 perp. height.

260000
,0005

130,0000 bushels, *Answer.*

2. What number of bushels in a conical heap of rye, the perpendicular height being 35 inches, and the slanting height 65 inches? —— *Ans.*, 52,5 bushels.

When grain is heaped against the side of the barn.

Rule. Multiply the difference of the squares of the heights by one half of the perpendicular height, and this product by ,0005; the result will be the content in bushels.

1. How many bushels of oats are in a heap, the perpendicular height being 30 inches, and the slanting height 60 inches? *Ans.*, 20,25 bushels.

2. How many bushels of beans are in a heap, the perpendicular height being 25 inches, and the slanting height 50 inches?

When grain is heaped in the corner of the barn.

Rule. Multiply the difference of the squares of the heights by one fourth of the perpendicular height, and this product by ,0005; the result will be the content in bushels.

1. Required the number of bushels of grain heaped in the corner of the barn; the perpendicular height being 40 inches, and the slanting height 70 inches? *Ans.*, 16,5.

2. How many bushels of barley in the corner of a box, the perpendicular height being 24 inches, and the slanting height 36 inches? *Ans.*, 2,16 bushels.

OF THE LEVER.

It is a principle in mechanics, that the power is to the weight as the velocity of the weight is to the velocity of the power.

To find what weight may be balanced by a given power.

Rule. As the distance between the body to be raised, or balanced, and the fulcrum, or prop, is to the distance between the prop and the point where the power is applied, so is the power to the weight which it will balance.

1. If a man weighing 160 lbs. rest on a lever 10 feet long, what weight will he balance on the other end, supposing the prop to be 1 foot from the weight?
1:9::160:1440 lbs. *Answer.*

2. If a weight of 1440 lbs. were to be raised by a lever 10 feet long, the prop being 1 foot from the weight, what power must be applied to the other end, to balance the weight? *Answer*, 160 lbs.

3. At what distance from the prop must a power of 160 lbs. be applied to balance 1440 lbs. 1 foot from the prop? *Answer*, 9 feet.

4. At what distance from a weight of 1440 lbs. must a prop be placed, so that a power of 160 lbs. applied 9 feet from the prop may balance it? *Answer*, 1 foot.

5. In giving directions for making a chaise, the length of the shafts between the axletree and back band being settled at 9 feet, a dispute arose whereabout on the shafts the centre of the body should be fixed. The chaise-maker advised to place it 30 inches before the axletree; others supposed that 20 inches would be a sufficient incumbrance for the horse. Now, supposing 2 passengers to weigh 3 cwt., and the body of the chaise $\frac{3}{4}$ cwt. more, what will the horse, in both these cases, bear more than his harness?

Answer, $\begin{cases} 116\frac{2}{3} \text{ lbs. in the 1st.} \\ 77\frac{7}{9} \text{ lbs. in the 2d.} \end{cases}$

OF THE WHEEL AND AXLE.

Rule. As the diameter of the axle is to the diameter of the wheel, so is the power applied to the wheel to the weight suspended on the axle.

1. If the diameter of the axle be 6 inches, and that of the wheel be 60 inches, what weight applied to the wheel will balance 10 lbs. on the axle? 60:6::10: 1 lb., *Ans.*

2. If the diameter of a wheel be 60 inches, and that of the axle be 6 inches, what weight on the axle will balance 1 lb. on the wheel? 6 : 60 : : 1 : 10 lbs., *Answer.*

3. If the diameter of the wheel be 60 inches, what must be the diameter of the axle, so that 1 lb. on the wheel may balance 10 lbs. on the axle? *Answer*, 6 in.

4. If the diameter of the axle be 6 inches, what must be the diameter of the wheel, so that 10 lbs. on the axle may balance 1 lb. on the wheel? *Answer*, 60 in.

OF THE SCREW.

The power is to the weight which is to be raised as the distance between two threads of the screw is to the circumference of a circle, described by the power applied at the end of the lever.

RULE.

Multiply twice the length of the lever by 3,1416, which will give the circumference of the circle; then say, as the circumference is to the distance between the threads of the screw, so is the weight to be raised to the power which will raise it.

1. The threads of a screw are 1 inch asunder; the lever, by which it is turned, is 30 inches long, and the weight to be raised is 1 ton,=2240 lbs.; what power must be applied to turn the screw? 30×2=60 and 60×3,1416= 188,496 inches, the circumference. Then 188,496 : 1 : : 2240 : 11,88 lbs., *Answer.*

2. If the lever be 30 inches, the circumference of the circle described by the power 188,496, the threads of the screw 1 inch asunder, and the power 11,88 lbs., what weight will be raised? *Answer*, 2240 lbs.

3. If the weight be 2240 lbs., the power 11,88 lbs., and the lever 30 inches in length, what is the distance between the threads of the screw? *Answer*, 1 inch.

4. If the power be 11,88 lbs., the weight 2240 lbs., and the threads 1 inch asunder, what is the length of the lever? *Answer*, 30 inches, nearly.

MISCELLANEOUS QUESTIONS.

1. Suppose a man to owe A, B and C, $1468; to A he owes $461, to B he owes $581; how much does he owe C? *Answer*, $426.

2. The less of two numbers is 2173, their difference is 371; what is the greater number? *Answer*, 2544.

3. A had 726 yards of cloth, after selling 340 yards; how many had he at first? *Answer*, 1066.

4. B started for Boston to purchase goods, having with him $1538: on his way, he had $706 stolen; how many dollars had he left? *Answer*, $832.

5. How many square rods in a piece of land, 72 rods long, and 19 rods wide? *Answer*, 1368 rds.

6. If a farm of 399 acres be divided into 21 lots, how many acres in a lot? *Answer*, 19 acres.

7. How many square chains in a field, 40 chains long, $38\frac{3}{4}$ wide? *Answer*, 1550 chains.

8. What will 765 lbs. of butter cost, at 21 cents per lb.? *Answer*, $160,65.

9. What number is that, which multiplied by 8 the product will be 25929? *Answer*, $3241\frac{1}{8}$.

10. What number is that, which multiplied by 15 will produce $\frac{3}{4}$? *Answer*, $\frac{1}{20}$.

11. What is the decimal equivalent to $\frac{1}{8}$? *Ans.*, ,125.

12. What fraction is that to which, if you add $\frac{1}{5}$, the sum will be $\frac{1}{2}$? *Answer*, $\frac{3}{10}$.

13. What fraction is that to which, if you add $\frac{4}{7}$, the sum will be $\frac{23}{28}$? *Answer*, $\frac{1}{4}$.

14. What fraction is that from which if $\frac{1}{4}$ be subtracted, the remainder will be $\frac{7}{36}$? *Answer*, $\frac{4}{9}$.

15. If $3\frac{1}{4}$ cwt. of coffee cost £9 9s. 7d., what is it per pound? *Answer*, $6\frac{1}{2}$d.

16. What will 8 cwt. of cheese cost, at £1 10s. 5d. per cwt.? *Answer*, £12 3s. 4d.

17. What is the sum of $\frac{1}{8}$ of a ton, and $\frac{15}{16}$ of a cwt.? *Answer*, 3 cwt. 1 qrs. 21 lbs.

18. What is the difference between $\frac{5}{8}$ of a mile, and $\frac{6}{7}$ of a league? *Answer*, 2 m. and $5\frac{5}{6}$ rds.

19. How much will 128 bushels of wheat cost, at 12s. 6d. per bushel? *Answer*, £80.

20. If £30 be divided equally among 27 men, how much would each receive? *Answer*, £1 2s. $2\frac{2}{3}$d.

21. If $\frac{1}{3}$ of 12 be 5, what will $\frac{1}{4}$ of 20 be? *Ans.*, $6\frac{1}{4}$.

22. If $\frac{1}{2}$ of $\frac{3}{7}$ of $\frac{7}{40}$ of $\frac{3}{5}$ of $\frac{5}{9}$ of a vessel be worth £378, how many dollars is $\frac{3}{32}$ of it worth? *Ans.*, $9450.

23. A person owning $\frac{2}{5}$ of a ship, sold $\frac{5}{8}$ of his share for $3750; what was the whole ship worth? *Answer*, $15000.

24. What is the sum of the third and half third of 3s. 4d.? *Answer*, 1s. 8d.

25. How many solid feet in a stick of timber 17 inches square, and 6 feet 5 inches long? *Answer*, 12 ft. 1517 in.

26. What in length, that is $4\frac{1}{2}$ inches in width, will make a square foot? *Answer*, 32 in.

27. A man owning $\frac{2}{5}$ of a farm, sold $\frac{1}{2}$ of his share for $245; what was the value of the farm? *Ans.*, $1225.

28. What is the interest of $936,21 for 11 weeks, at 6 per cent.? *Answer*, $11,914.

29. What will $$400\frac{1}{4}$ amount to in 16 years, at 6 per cent. simple interest? *Answer*, $784,49.

30. What is the interest of $300,87, for 1 month and 15 days, at 9 per cent.? *Answer*, $3,384.

31. What is the interest of £300, for $5\frac{3}{4}$ years, at $3\frac{3}{4}$ per cent.? *Answer*, £64 13s. 9d.

32. A holds B's note for $2000, dated June 1st, 1825, on which are the following endorsements—viz:

Received	Sept. 1st,	1825	$96
	Dec. 10th,	1825	$15
	April 20th,	1826	$36
	July 1st,	1826	$200
	Jan. 10th,	1827	$20
	March 25th,	1827	$90

How much remains due, June 1st, 1827? *Answer*, $1767,48.

33. What principal, at 5 per cent., will amount to $725 in 9 years? *Ans.*, $500.

34. What principal will gain $150 in 1 year, at 6 per cent.? *Ans.*, $2500.

35. What is the present worth of $590, due 3 years hence, discounting at 6 per cent.? *Ans.*, $500.

36. What is the present worth of $200; $100 payable in 2 months, $50 in 3 months, and $50 in 5 months, discounting at 4 per cent.? *Ans.*, $197,05.

37. A note of $500 amounted to $725 in 9 years; what was the rate per cent.? *Ans.*, 5 per cent.

38 In what time will £420 amount to £520 16s., at 3 per cent.? *Ans.*, 8 years.

39. What will $1350 amount to in 3 years, at 5 per cent., compound interest? *Ans.*, $1562,793.

40. A has 150 gallons of wine, which he will sell at 7s. 3d. per gallon, ready money, but in barter he will have 8s. per gallon. B has linen at 3s. 6d. per yard, ready money; how must B sell his linen per yard, in proportion to the bartering price of A's wine, and how many yards will be equal to A's wine?

Ans., { Bartering price, 3s. 10¼d,
and 310 yards,2 qrs. 3 na.

41. A merchant bought hats at 4s. each, and sold them at 4s. 9d.; what was the gain in laying out £100? *Ans.*, £18 15s.

42. A merchant bought 10 tons of iron for £200. The freight and duties amounted to £25, and his own charges to £8 6s. 8d.; for how much per pound must he sell it to gain 20 per cent.? *Ans.* 3d. per lb.

43. Sold a watch for £50, and by so doing lost 17 per cent., whereas I ought to have cleared 20 per cent. How much was it sold under its real value? *Ans.*, £20 5s. 9¼d.

44. Four men trade in company. A's stock was $560; B's $1040; C's $1200; their whole stock was $3200. They gained in two years a sum equal to twice their stock, and $160 more; what was D's stock, and what was each man's share of the gain?

Ans., { D's stock, $400.
A's gain, $1148,
B's " $2132,
C's " $2460,
D's " $ 820.

45. A, B and C trade in company. A put in $20, B $30, and C a sum unknown; the gain was $36, of which $16 was C's share; what was C's stock, and what was A's and B's gain?

Ans., { C's stock, $40,
A's gain, $ 8,
B's " $12.

46. What is the square root of 42¼? *Ans.*, 6½.

47. What is the mean proportional between 24 and 96? *Ans.*, 48.

48. A certain general has an army of 5625; how many must he place in rank and file, to form them into a square? *Ans.*, 75.

49. Arrange 10952 men in such a manner, that the number in rank may be double the file? *Ans.*, 74 in file, and 148 in rank.

50. There is a circle whose diameter is 4 inches; what is the diameter of a circle 3 times as large? *Ans.*, 6,928+.

51. Two boats start on a river at the same time, from places 300 miles apart; the one proceeding up the stream is retarded by the current 2 miles per hour, while that moving down the stream is accelerated 2 miles per hour; both are propelled by a steam engine, which would move them in still water 8 miles per hour. How far from each starting place will the boats meet? *Ans.*, 112½ miles from the lower place, and 187½ miles from the upper place.

52. There are 3 circular ponds; the diameter of the less is 100 feet; the diameter of the greater is 3 times as large; what is its diameter? *Ans.*, 173,2+.

53. What is the superficial content of one side of a cubical stone, containing 474512 solid inches? *Ans.*, 6084 inches.

54. If a cube of silver, whose side is 4 inches, be worth £50, what is the side of a cube of like silver, worth 4 times as much? *Ans.*, 6,349 inches.

55. The height of a tree standing on the bank of a river is 75 feet; a line reaching from the opposite shore to the top of the tree is 256 feet long. What is the breadth of the river? *Ans.*, 244,7+ feet.

56. If a pipe 6 inches in diameter will discharge a certain quantity of water in 4 hours, in what time will 3 pipes, each 4 inches diameter, discharge double the quantity? 10⅔ hours.

56. Two men start from the same place and travel, one south 76 leagues, the other east 45 leagues. How far will they be apart? *Ans.*, 88,3+ leagues.

58. What is the side of a cubical mound containing 5832 solid feet? *Ans.*, 18 feet.

59. If a ball, 6 inches in diameter, weigh 32 lbs., how much will a ball of the same metal weigh, whose diameter is 3 inches? *Ans.*, 4 lbs.

60. The side of a cubical box is 2 feet; what is the side of a box which will contain 3 times as much?

Ans., $10\frac{3}{5}$ inches.

61. A refiner mixed 3 lbs. of gold, 22 carats fine, with 3 lbs. 20 carats fine; what was the fineness of the mixture?

Ans., 21 carats.

62. How many gallons of water must be put to wine at 3s. a gallon, to fill a vessel of 100 gallons, so that a gallon of the mixture may be afforded at 2s. 6d. per gallon?

Ans., $16\frac{2}{3}$ gallons.

63. If 12 bushels of oats, at 1s. 6d. per bushel, be mixed with barley at 2s. 6d., rye at 3s., and wheat at 4s. per bushel, how much barley, rye and wheat, must be mixed with the 12 bushels of oats, that the mixture may be worth 2s. 9d. per bushel? *Ans.*, 12 bushels of each sort.

64. A man travelled 6 miles the first day, 9 the second, increasing each day's journey 3 miles—he travelled 61 days; how many miles did he travel the last day?

Ans., 186 miles.

65. If 100 pears be placed in a right line, 1 yard asunder, how many miles will a man travel, to pick them and carry them one at a time to a basket placed 1 yard from the first pear? *Ans.*, 5 mi.—1300 yds.

66. Suppose 550 men are in a garrison, with provision sufficient to last them 9 months; how many must depart, that their provision may last them 11 months?

Ans., 100.

67. If a man labor for me 16 days, when the price of labor is $1,25, how long must I labor for him, to requite the favor, when the price of labor is 75 cents per day?

Ans., $26\frac{2}{3}$ days.

68. The third part of an army were killed, the fourth part were taken prisoners, and 1000 fled; how many were in this army? *Ans.*, 2400.

69. An ignorant fop, wanting to purchase an elegant house, met with a facetious gentleman, who told him he had one which he would sell him on the following very reasonable terms, viz:—that he should give him 1 penny for the first door, 2 for the second, 4 for the third, and so on, doubling the price of each door, which were 36 in number. It is a bargain, exclaimed the simpleton, and here is a guinea to bind it; what did the house cost him?

Ans., £286331153 1s. 3d.

70. A, B and C would divide $100 between them, so that B may have $3 more than A, and C $4 more than B. What is each man's share?

Answer, A's $30,
B's $33,
C's $37.

71. Divide £340 among 3 men, in such a a manner that the first shall have 3 times as much as the second, and the third 4 times as much as the first?

Answer, 1st £$63\frac{3}{4}$,
2d £$21\frac{1}{4}$,
3d £255.

72. A person having a certain number of dollars, said, if a third, a fourth, and a sixth of them were added, their number would be $45. How many dollars had he?

Answer, $60.

73. What number is that which, being multiplied by $\frac{3}{4}$, the product will be $15\frac{3}{4}$? *Answer*, 21.

74. What number is that from which, if you subtract $\frac{2}{5}$ of itself, the remainder will be 12? *Answer*, 20.

75. What part of 25 is $\frac{5}{8}$ of a unit? *Answer*, $\frac{1}{40}$.

76. What number is that which, being multiplied by $\frac{2}{3}$, the product will be $\frac{1}{4}$? *Answer*, $\frac{3}{8}$.

77. If $\frac{3}{8}$ of a farm be worth £3740, what is the whole worth? *Answer*, £9973 6s. 8d.

78. A father dying, left his son a fortune, $\frac{3}{16}$ of which he spent in 6 months; $\frac{2}{3}$ of the remainder lasted him 12 months longer, when he had £348 left. What sum did he receive? *Answer*, £1284 18s. $5\frac{7}{13}$d.

79. A young man received $210, which was $\frac{2}{3}$ of his elder brother's fortune, and 3 times the elder brother's fortune was $\frac{1}{2}$ the father's estate. What was the value of the estate? *Answer*, £1890.

80. A man has 80 shillings to divide among his laborers, consisting of an equal number of men, women and boys. To every boy he gives 6d., to every woman 8d., to to every man 1s. 4d. How many were there of each?

Answer, 32.

81. Suppose a man pay to his laborers, men, women and boys, £7 17s. 6d.; to every boy he gave 6d.; to every woman 8d., and to every man 16d.; for every boy there were 3 women, and for every woman there were 2 men. How many were there of each?

Ans., 15 boys, 45 women and 90 men.

82. A gentleman bought a horse, a chaise and harness, for £60. The horse cost twice as much as the chaise, and the harness half as much as the horse. What was the cost of each?

Answer, horse £30,
chaise £15,
harness £15.

83. Divide $1000 among 3 men, in such a manner that as often as the first has $3, the second shall have $5 and the third $8?

Answer, 1st $187,50,
2d $312,50,
3d $500.

84. A can do a piece of work in 10 days; D can do the same in 13 days. In what time will both working together do the same work? *Answer*, $5\frac{15}{23}$ days.

85. If 6 lbs. of pepper be worth 13 lbs. of ginger, and 19 lbs. of ginger be worth $4\frac{3}{4}$ lbs. of cloves, and 10 lbs. of cloves be worth 63 lbs. of sugar, at 10 cents per pound, what is the value of 1 cwt. of pepper? *Ans.*, $38,22.

86. A tradesman increased his estate annually one third, less $960 which he spent in his family. At the end of $3\frac{1}{4}$ years, he found that his estate amounted to $30284. What had he at first? *Answer*, $13551,75.

87. A and B have the same income. A saves $\frac{1}{8}$ of his, but B, by spending $30 per annum more than A, at the end of 8 years finds himself $40 in debt. What was their income, and what did each spend per annum?

Answer, Their income $200.
A spends $175,
B spends $205.

88. What number is that which, being increased by its $\frac{1}{2}$, $\frac{1}{4}$ and 5 more, the number will be double?

Answer, 20.

89. A man, after having spent $\frac{1}{2}$ and $\frac{1}{3}$ of his money, had £$26\frac{2}{3}$ left. How much had he at first?

Answer, £160.

90. A vessel has 3 pipes; the first will fill it in $\frac{1}{2}$ of an hour, the second in $\frac{1}{3}$ of an hour, and the third in $\frac{1}{4}$ of an hour. In what time will all running together fill it?

Answer, $\frac{1}{9}$ of an hour.

91. A and B employed equal sums in trade. A gained a sum equal to $\frac{1}{4}$ of his stock; B lost $225; then A's money was double that of B's. What was each man's stock? *Answer*, $600.

92. There is a certain number, $\frac{7}{8}$ of which exceeds $\frac{4}{5}$ by 6. What is that number? *Answer*, 80.

93. If $60 be divided between 4 men in such a proportion, that the first shall have $\frac{1}{3}$, the second $\frac{1}{4}$, the third $\frac{1}{5}$, and the fourth $\frac{1}{6}$, what will each receive?

Answer,
1st \$$21\frac{1}{19}$,
2d \$$15\frac{15}{19}$,
3d \$$12\frac{12}{19}$,
4th \$$10\frac{10}{19}$.

94. A, B, C and D spent 35 shillings at a reckoning, and being a little *dipped*, agreed that A should pay $\frac{2}{3}$, B $\frac{1}{2}$, C $\frac{1}{3}$ and D $\frac{1}{4}$. What does each pay in this proportion?

Answer,
A's 13s. 4d.,
B's 10s.,
C's 6s. 8d.,
D's 5s.

95. A laborer was hired 60 days upon these conditions: that for every day he worked he should receive 3s. 4d., and for every day he was idle he should forfeit 1s.8d. At the expiration of the time, he received £3 15s. How many days did he work, and how many was he idle?

Answer, he worked 35 days, and was idle 25.

96. A, B and C traded in company, and gained £350, of which A took a certain sum; B took 4 times as much as A, and C 8 times as much as B. What were their respective shares of the gain?

Answer,
A's gain £9 9s. 2d. $1\frac{3}{37}$qr.,
B's " £37 16s. 9d. $0\frac{12}{37}$qr.,
C's " £302 14s. 0d. $2\frac{22}{37}$qrs.

97. A gentleman divided his fortune among his sons, giving A £9 as often as B £5, and C £3 as often as B £7 C's dividend was £$1537\frac{5}{8}$. To what did the whole estate amount? *Answer*, £11583 8s. 10d.

98. If $\frac{5}{7}$ of $\frac{3}{8}$ of $\frac{4}{5}$ of a ship be worth $\frac{2}{9}$ of $\frac{7}{8}$ of $\frac{12}{13}$ of the cargo, valued at £1000, what is the value of both ship and cargo? *Answer*, £1837 12s. $1\frac{75}{117}$d.

99. Three men purchase a lot of land in company. A paid $\frac{2}{5}$; B $\frac{3}{7}$ of the whole, and C paid £256. How much did A and B pay, and what part of the land had C?

Answer,
A paid £597 6s. 8d.;
B " £640;
C's share $\frac{6}{35}$.

100. A gay fellow soon got the better of $\frac{2}{7}$ of his fortune. He then gave £1500 for a commission, and his profusion continued until he had but £450 left, which he found to be just $\frac{3}{8}$ of his money after purchasing his commission. What was his fortune at first? *Answer*, £3780.

101. A and B are on opposite sides of a circular field, 268 rods in circumference; they start both at the same time to go round it, and go the same way. A goes 22 rds. in 2 minutes; B goes 34 rods in 3 minutes. How many times will they go round the field before B will overtake A? *Answer*, 17 times.

102. A man driving his geese to market, was met by another, who said, "Good morrow, sir, with your hundred geese." Said he, "I have not a hundred, but if I had, in addition to my present number $\frac{1}{2}$ as many as I now have, and two geese and a half, I should have a hundred." How many had he? *Answer*, 65 geese.

103. The hour and minute hand of a watch are exactly together at 12 o'clock. When are they next together? *Answer*, 1 h. 5 m. $27\frac{3}{11}$s.

104. Three men trade in company till they gain £120; the sums put in were in such proportion that as often as A had £5 of the gain B had £7, and as often as B had £4 C had £6. What was each man's share of the gain?

Answer, A's £26 13s. 4d.;
B's £37 6s. 8d.
C's £56

105. A and B cleared by an adventure at sea 45 guineas, which was £35 per cent. upon money adventured. With this gain they agreed to purchase a genteel horse and carriage, which they were to use in proportion to their sums adventured, which was found to be 11 to A as often as 8 to B. What money did each adventure?

Answer, A £104 4s. $2\frac{10}{19}$d.
B £75 15s. $9\frac{9}{19}$d.

106. A military officer drew up his soldiers in rank and file, having the number in rank and file equal. On being reinforced with three times his first number of men, he placed them all in the same form, and then the number in rank and in file was just double what it was at first. He was again reinforced with 3 times his whole number of men; and, after placing the whole in the same form as at first, his number in rank and in file was 40 men each. How many men had he at first? *Answer*, 100 men.

107. A general disposing his army into a square battalion, found he had 231 over and above, but increasing each side with one soldier, he wanted 44 to fill up the square. Of how many men did his army consist? *Ans.*, 19000.

108. There are 3 horses belonging to different men, employed to draw a load of salt from Boston to Lowell for $\$9,50$. A's and B's horses are supposed to do $\frac{7}{8}$ of the work, A's and C's $\frac{7}{12}$, B's and C's $\frac{17}{24}$. They are to be paid proportionally. What is each man's share of the gain?

Answer, A's $\$3,288\frac{6}{13}$,
B's $\$4,384\frac{8}{13}$,
C's $\$1,826\frac{12}{13}$.

109. A, B and C are to share £100 in the proportion of $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ respectively, but C dying, it is required to divide the whole sum properly between the other two.

Answer, A's share £57 2s. 10$\frac{1}{4}$d.
B's " £42 17s. 1$\frac{3}{4}$d.

110. There is an island 50 miles in circumference, and 3 men start together to travel the same way round it. A travels 7 miles a day, B 8, and C 9. When will they all come together again, and how far will each travel?

111. A man died leaving $1000 to be divided between his two sons, one 14 and the other 18 years of age, in such a manner that the share of each being let, at 6 per cent. interest, should amount to the same sum when they should arrive at the age of 21. What did each receive?

Answer, The eldest $\$546,153+$.
The youngest $\$453,846+$.

112. A hare starts 12 rods before a greyhound, but is not perceived by him till she has been up 45 seconds. She scuds away at the rate of 10 miles an hour, and the dog, on view, makes after, at the rate of 16 miles an hour. How long will the course hold, and what space will be run over from the spot where the dog started?
97$\frac{1}{2}$ seconds.
2288 feet.

113. There is a circular field surrounded by a rail fence 10 rails high, each 1 rod in length, and the number in the fence equals the number of acres in the field. What is the area of the field? *Answer*, 20106,176 acres.

114. How much greater is the circle described by the head of a man, 6 feet high, than his feet, in the revolution of the earth on its axis? *Answer*, 37,69+ feet.

115. In an orchard, $\frac{1}{2}$ the trees bear apples, $\frac{1}{4}$ pears, $\frac{1}{6}$ plums, and 50 of them produce nothing. How many of the trees are there in all? *Answer*, 600.

116. Sound moves at the rate of 1142 feet in a second. If the time between the lightning and the thunder be 30 seconds, what is the distance of the explosion?
Ans., 6,488+ miles.

117. In a thunder storm I observed by my watch that it was 6 seconds between the lightning and the thunder. At what distance was the explosion? *Ans.*, 6852 feet.

118. Tubes may be made of gold, weighing not more than at the rate of $\frac{1}{1625}$ of a grain per foot. What would be the weight of such a tube, which would extend across the Atlantic, from Boston to London, the distance being 3000 miles? *Ans.*, 1 lb. 8 oz. 6 pwt. $3\frac{9}{13}$ grs.

119. Suppose one of those meteors, called fireballs, to move parallel to the earth's surface, and 50 miles above it, at the rate of 20 miles per second, in what time would it move round the earth?

The earth's diameter being 7964 miles, the diameter of the orbit will be $7964+50\times2=8064$, and $8064\times3{,}1416=25333{,}8624$, its circumference. Then $25333{,}8624\div20=1266{,}693120=21'\ 6''\ 41'''\ 35''''\ 13'''''\ 55''''''$, the *answer*.

BOOK-KEEPING.

All mercantile transactions consist in exchanging articles of trade, either for money or its equivalent. A systematic record of such transactions is called Book-Keeping. Every person should possess sufficient knowledge of this science to keep such record of his business as will at any time exhibit a true state of his affairs.

The person who purchases goods, or receives any thing of me, is debtor to me; and he who pays me money, or receives any thing of me, is creditor.

The following is a plain and simple method of keeping accounts without a day-book, and will be found sufficient for the purposes of farmers and mechanics, where their business is such, that charges are made only at considerable intervals; but in all cases where some three or four, or more charges are made daily, a day-book should be regularly kept.

1837.	James Trueworthy Dr.	$	C	1837.	James Trueworthy Cr.	$	C
Jan. 1,	To 4 cords of wood at $3	12	00	Jan. 1,	By 3 gals. molasses, at 40 cts.	1	20
Jan. 9,	To 1 load of hay at $20	20	00	March 1,	By 1 lb. of starch, at 12 cts.		12
Feb. 2,	To 20 bushels of oats at 50 cts.	10	00	April 2,	By 50 lbs. of coffee at 14 cts.	7	00
June 1.	To 15 lbs. of veal at 5 cts.		75	June 13,	By 12 lbs. sugar at 9 cts.	1	08
July 6,	To pasturing horse 3 weeks, at 50 cts.	1	50	" 20,	By 3 yds. of broad cloth, at $5	15	00
				Aug. 19,	By cash to balance	19	85
	$	44	25		$	44	25

BOOK-KEEPING BY SINGLE ENTRY.

By Single Entry two principal books are required: the *Day*, or *Waste-Book*, and *Leger*.

The day-book should be ruled with two columns on the right hand, for dollars and cents, and one column on the left, for noting the month, day, and year, when the charges were made.

All charges should be made at or near the time when they bear date, or when they purport to have been made. Where an individual is charged with articles delivered to him, he is a debtor for the amount of them, and against his name on the day-book, "Dr." is written, to designate him as debtor. When such individual delivers articles to be allowed him on his account, he is said to be a creditor, and "Cr." is written against his name on the day-book, to designate that he is credited with such articles.

THE LEGER.

The Leger is a book to which the accounts entered on the Day-Book are transferred from time to time, in order that each man's whole account may appear by itself. All articles for which he is debtor are usually entered on the left hand side of the page, and all articles for which he is creditor, are entered on the right hand side of the page.

The date of the charge, and the amount in dollars and cents, are entered in the same manner as on the Day-Book, but where there are a number of charges of different articles made under one date on the Day-Book, they are usually entered upon the Leger, as " Sundries," which is a general term for a variety of articles, and the total amount of all such articles charged on any one day, are carried out against such entry, in dollars and cents.

When a charge is posted from the Day-Book to the Leger, some bracket or other mark, as in the account posted, is made on the left of the charge on the Day-Book, to denote that it has been transferred to the Leger.

When the account is to be settled, the sum requisite to balance the same is ascertained, and cash, or a note is given, which is entered upon the book, and the account is thus made equal. Traders and mechanics most usually give receipted bills of their accounts on settlement. In some instances, a settlement is written at the bottom of the account, as follows :—" Settled and balanced all accounts to this date," which is signed by the parties.

FORM OF THE DAY-BOOK.

1837. Jan'y 1,	John Newton Dr.		
{	To 4 gallons Molasses at 40 cts.	1	60
	15 lbs. of Sugar at 10cts.	1	50
	Joel Stephenson Dr.		
	To 6 lbs. chocolate at 1s.	1	00
	Jedidiah Jones Dr.		
	To 50 lbs Fish at 4 1-2cts	2	25
	2 lbs. Starch at 14 cts.		28
	6 lbs. Coffee at 15 "		90
" 3,	John Silvers Dr.		
	To 38 lbs. Rice at 5 cts.	1	90
	2 " Cocoa at 28 cts.		56
	Joel Mason Dr.		
	To 50 lbs. of iron at 7cts.	3	50
	1 barrel of flour	7	50
" 4,	Timothy Styles Dr.		
	To 6 gals. of Oil at 6s.	6	00
	50 lbs. Fish at 5 1-2cts.	2	25
" 5,	John Silvers Dr		
	To 40 lbs. of Raisins at 14 cts.	5	60
{	John Newton Dr.		
	To 1 gal. Oil	1	00
	8 lbs. Coffee at 15	1	20
	Cr. by Cash 10s. 6d.	1	75
	Joel Mason Dr.		
	To 3 bush. of Salt at 6s.	3	00
{	John Newton Dr.		
	To 2 lbs. Salaratus at 11 cts.		22
1837. Jan. 6,	John Silvers Dr.		
	To 4 yds. Broadcloth at 4,50	18	00
	Cr. by 8 bush. of rye at 4s. 6d.	6	00
	By 15 lbs. dried apples at 5 cts.		75
	40 lbs. Cheese at 10cts.	4	00
" 7, {	John Newton Dr.		
	To 3 yds. Flannel at 3s.	1	50
	Joel Stephenson Dr.		
	To 1 bush. Corn at 9s.	1	50
	1 Fur Hat	4	00
" 8, {	John Newton Cr.		
	By 1 Ton of Hay	15	00
	Dr. To 1 bl. Flour	7	50
	To 50 lbs. Sugar	5	00
	Jedidiah Jones Dr.		
	To 38 lbs. Rice at 5 cts.	1	90
	Joel Mason Dr.		
	To 1 lb. Y. H. Tea 3s.		50
	1 bush. Salt	1	00
	1 lb. Cocoa		28
	John Silvers Dr.		
	To 1 pr. of Boots	3	50
	1 " Shoes	2	00
	Timothy Styles Dr.		
	To 100 lbs. Nails at 6cts.	6	00
	Joel Stephenson Dr.		
	To 40 lbs. Butter at 20cts.	8	00
	1 Firkin 3s.		50

1837.	John Newton Cr.		
Jan. 10.	By 40 lbs. dried apples	2	00
	Dr. To 2 lb.s Y. H. Tea at 3s. 6d.	1	16
	To 2 lbs. Raisins 2s.		34
" 11.	Jedidiah Jones Dr.		
	To 2 1-2 gal. Oil 6s.	2	50
	Joel Mason Cr.		
	By 5 bush. of apples 2s.	1	67
	1 1-2 cords wood at 15s.	3	75
	Cash to balance ac,t.	10	44
	John Silvers Cr.		
	By 30 bush. potatoes at 1s. 6d.	7	50
	100 lbs. Cheese	10	00
" 12.	Jedidiah Jones Cr.		
	By Cash rec'd of I. Hardy	3	00
	2 day's labor at 4s. 6d.	1	50
	Cash to balance ac't	3	33

1837.	John Silvers Cr.		
Jan. 12	By 1 day's work self & cattle	2	00
	3 cords of wood at 15s.	10	00
	Dr. To 1 barrel of Flour	7	50
" 13.	Joel Stephenson Cr.		
	By 2 cords of Bark at 12s.	4	00
	Timothy Styles Cr.		
	By 2 Firkins Butter 80 lbs. at 1s.	13	34
	" Cash		91
" 14.	John Newton Dr.		
	To 100 lbs. Fish at 5 cts.	5	00
	Cr. By 2 cords of wood at 15s.	5	00
	By Cash to bal. ac't	2	77
	John Silver Dr.		
	To 100 lbs. iron at 7 cts.	7	00
" 15.	Joel Stephenson Cr.		
	By 10 lbs. lard at 1s.	1	67
	20 lbs. Butter at 20cts.	4	00

FORM OF THE LEGER.

JOHN NEWTON, Dr. Cr.

1837.				1837.			
Jan'y 1,	To 4 gal. Molasses at 40cts.	1	60	Jan. 5.	By Cash,	1	75
	" 15lbs. Sugar at 10cts.	1	50	" 8.	By 1 ton of hay,	15	00
" 5,	To Sundries,	2	42	" 14.	By 2 cords of wood at 15s.	5	00
" 7,	" 3 yds. Flannel at 3s.	1	50	" "	Cash to balance accounts.	3	27
" 8,	" Sundries,	12	50			$ 26	02
" 10,	To Sundries,	1	50				
" 14,	" 100 lbs. Fish at 5 cts.	5	00				
		$26	02				

JOEL STEPHENSON, Dr. Cr.

1837.				1837.			
Jan. 1,	To 6 lbs. chocolate at 1s.	1	00	Jan. 13,	By 2 cords of wood at 12s.	4	00
" 7,	" Sundries,	5	50				

JEDIDIAH JONES, Dr. Cr.

1837.				1837.			
Jan'ry 1,	To sundries,	3	43	Jan. 12,	By sundries,	4	50

JOHN SILVERS, Dr. Cr.

1837.				1837.			
Jan'ry 3,	To sundries,	2	46	Jan. 6,	By sundries,	10	76

JOEL MASON, Dr. Cr.

TIMOTHY STYLES, Dr.				Cr.		

The student should be required to complete posting the above accounts from the Day Book.

To each Leger there is an Alphabet, or Index of Names, consisting of a small book with the letters of the alphabet pasted upon the leaves, so that each man's name can be readily found by turning to the leaf marked by the first letter of his sir-name.

INDEX TO THE LEGER.

	PAGE.				
A		H		O	
Allen James	1	Holt William	8	Odlin Woodbridge	16
B		Horsley Henry	9	P	
Bacon Samuel	2	J		Pratt Thomas	17
Brown Leonard	3	Jenkins Abraham	9	Putney Andrew	18
C		Jones Jedidiah	10	R	
Canning George	3	K		Ramsay Amos	19
D		Knowles Michael	11	Rollins Zenas	20
Dalton Levi	4	L		S	
E		Leeds William	12	Stephenson Joel	21
Edgell Oliver	4	M		Silvers John	22
F		Mason Joel	13	Styles Timothy	23
Fulton Curtis	5	M'Farland Asa	13	T	
Freeman Nathan	6	N		Trueworthy James	24
G		Newton John	14	W	
Grosvenor Jacob	7	Nelson John	15	Wyndham Augustus	25

Every person, engaged in business, should take an Inventory of his notes and accounts, his stock in trade, and other property, and of the debts owed by him, once or twice a year. By comparing this with former Inventories, he will know his gain or loss from time to time.

Inventory of Stock in Trade, Notes, Accounts, and other property, taken from the foregoing example, July 1, 1837.

200 lbs. Coffee, at 1s.	$	33	34	Am't brought up, $	567	34
2 kegs Tobacco—100 lbs. at 20 cts.		20	00	Note against James Allen, for $50 and int., dated Jan. 1, 1836,	53	00
5 bales Cotton—1500 lbs.	10 cts.	150	00	Note against Thomas Pratt, for $300, dated July 1, 1836, and interest,	309	00
3 boxes Sugar—1200 lbs.	9 cts.	108	00			
4 casks Nails—1600 lbs.	6 cts.	96	00			
40 bushels Corn, at 6s.		40	00	Bal. on ac't against John Silvers	5	81
6 tons Hay, at $10,		60	00	do. against Joel Stephenson,	5	33
400 lbs. Iron, at 6 cts.		24	00	1 Horse,	75	00
90 galls. Molasses, at 40 cts.		36	00	1 Chaise,	125	.
				Homestead,	1000	00
Am't carried up,	$	567	34			
					2140	48
				Inventory taken Jan. 1, 1837,	1950	75
				Net gain in 6 months,	179	73

HABITS OF A MAN OF BUSINESS.

A sacred regard to justice, forms the basis of every transaction, and regulates the conduct of the upright man of business. He is strict in keeping his engagements—does nothing carelessly, or in a hurry—employs nobody to do what he can easily do himself—keeps every thing in its proper place—leaves nothing undone which ought to be done, and which circumstances permit him to do; keeps his designs and business from the view of others—is prompt and decisive with his customers, and does not over-trade for his capital,—prefers short credits to long ones, and cash to credit transactions at all times, when they can be advantageously made, either in buying or selling; and small profits in credit cases, with little risk, to the chance of greater gains with more hazard. He is clear and explicit in all his bargains—leaves nothing of consequence to memory, which he can and ought to commit to writing; keeps copies of all his important letters which he sends away, and every letter, invoice, &c., belonging to his business, well classed, and put up in order: never allows his desk to be confused with many papers lying upon it,—is always at the head of his business, well knowing, that if he leave it, it will leave him: holds it a maxim, that he whose credit is suspected, is not safe to be trusted; is constantly examining his books, and sees through his affairs as far as care and attention enable him,—balances regularly at stated times, and then makes out and transmits all his accounts current to his customers and constituents, both at home and abroad;—avoids, as much as possible, all sorts of accommodation in money matters, and lawsuits, where there is the least hazard; is economical in his expenditure, always living within his income—keeps a memorandum book with a pencil in his pocket, in which he notes every particular relative to appointments, addresses, and petty cash matters,—is cautious how he becomes security for any person, and is generous only when urged by motives of humanity and principle.

OF NOTES AND BONDS.

No. I. Orford, August 10, 1837.

For value received, I promise to pay to John True, or order, seventy-five dollars, fifty cents, on demand, with interest. JOSEPH DENMAN.

Attest: Joel Trusty. —

No. II. Concord, July 4, 1837.

For value received, I promise to pay to James Doughty, or bearer, ten dollars, thirty-four cents, six months after date. — JOHN MORSE.

No. III. *By two persons.*

New-Haven, Oct. 6, 1837. For value received, we jointly and severally promise to pay to S. T., or order, seventeen dollars and eighty-eight cents, on demand, with interest.

OLONZO FONTAINELLE,
JAMES WHITEHEAD.

Attest: Timothy Trusty.

☞ **For Bank Notes, see "Discount," p. 171.**

No. I. *Of Receipts.*

NORWICH, JUNE 7, 1837. Received of Mr. NICHOLAS JEWETT five dollars, in full of all account.

HENRY SLOCUM.

No. II. *Receipt for an endorsement on a note.*

NEW-YORK, SEPT. 9, 1837. Received of Mr. JOHN HADLEY, (by the hand of James True) twenty-five dollars, fifty cents, which is endorsed on his note of May 6, 1836. PETER TRUSTY.

No. III. *Receipt for money received on account.*

MOUNT HOLLY, DEC. 9, 1837. Received of Mr. JOHN VONDYKE, twenty dollars on account.

THOMAS BEAN.

No. IV. *Receipt for interest due on a bond.*

Received, this fourteenth day of May, of Mr. S. W., the sum of six dollars, in full of one year's interest of 100 dollars due to me on the 16th day of April last, on bond from the said S. W. By me, C. B.

No. I. *Of Orders.*

Mr. JOEL McKNIGHT: Sir,

For value received, pay to O. S., ten dollars, and place the same to my account.

HOOKSETT, SEPT. 7, 1837. SUEL RYNO.

No. II. LOWELL, OCT. 10, 1837.

Sir,—For value received, pay S. O. twenty cents, and this with your receipt shall be your discharge from me. JUSTUS PRUDENS.

To Mr. DANIEL HOLDEN.

FORM OF A WARRANTEE DEED.

KNOW all men by these Presents, that I, A. B., of C., in the county of R., and State of New-Hampshire, printer, for and in consideration of the sum of one hundred and twenty dollars to me in hand before the delivery hereof, well and truly paid by D. E., of L., in the county of M., in said State, husbandmen, the receipt whereof I do hereby acknowledge, have gived, granted, bargained and sold, and by these presents do give, grant, bargain, sell, alien, enfeoff, convey and confirm unto the said D. E., his heirs and assigns forever, a certain tract of land, situate in said C., containing one hundred acres, more or less, bounded southerly on the highway leading from C. to H., easterly on land of G. H., northerly on land of I. R., and westerly partly on land of S. M., and partly on land of L. O. To have and to hold the said granted premises, with all the privileges and appurtenances to the same belonging, to him, the said D. E., and his heirs and assigns, to his and their only proper use and benefit forever. And I, the said A. B., and my heirs, executors and administrators, do hereby covenant, grant and agree, to and with the said D. E., and his heirs and assigns, that until the delivery hereof I am the lawful owner of the said premises, and am seized and possessed thereof in my own right in fee simple, and have full power and lawful authority to grant and convey the same in manner aforesaid; that the said premises are free and clear from all and every incumbrance whatsoever; and that I and my heirs, executors and administrators, shall and will warrant and defend the same to the said D.E. and his heirs and assigns, against the lawful claims and demands of any person or persons whomsoever.

Condition of Mortgage.

The condition of the foregoing deed is such, that if the said A. B. and E. F., their heirs, executors and administrators, shall well and truly pay unto the said G. H., his heirs and assigns, the sum of three hundred and fifty dollars, in three years, with interest annually, agreeably to their note, bearing even date herewith, then the foregoing deed is to be void and of no effect; otherwise, to remain in full force.

And I, M. B., wife of the said A. B., in consideration aforesaid, do hereby relinquish my right of dower in the before mentioned premises.

In witness whereof, we have hereunto set our hands and seals, this tenth day of May, in the year of our Lord one thousand eight hundred and thirty-seven.

Signed, sealed and delivered, } A. B. { SEAL }
in presence of us, }
P. G. M. B. { SEAL }
A. S.

State of New-Hampshire, Rockingham SS., C——, May tenth, 1837. Personally appearing the abovenamed

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Note. By omitting the condition of mortgage the Deed is the common form of a Warrantee Deed.

A. B., acknowledged the foregoing instrument to be his voluntary act and deed: Before me—

J. L., *Justice of Peace.*

FORM OF A QUITCLAIM DEED.

KNOW all men by these Presents, that I, A. B., of C., in the county of M., and State of New-Hampshire, yeoman, and M. B., wife of the said A. B., for and in consideration of one hundred dollars, to us in hand before the delivery hereof, paid by L. M., of G., in the county of H., in said State, wheelwright, have remised, released, and forever quitclaimed, and by these presents do remise, release, and forever quitclaim unto the said L. M., his heirs and assigns forever, a certain tract of land, situate in said G., containing one hundred acres, more or less, being Lot No. 16, in the sixth range and first division of lots in said G., laid out to the original right of J. S. To have and to hold the said remised premises, with all the privileges and appurtenances thereunto belonging, to him, the said L. M., his heirs and assigns forever; and we do hereby covenant with the said L. M., that we will warrant and defend the said premises to him, the said L. M., his heirs and assigns, against the lawful claims and demands of any person or persons claiming by, from, or under us.

In witness whereof, we have hereunto set our hands and seals, this tenth day of May, in the year of our Lord one thousand eight hundred and thirty-seven.

Signed, sealed and delivered in presence of A. B. { SEAL }
J. D.
J. S. M. B. { SEAL }

STATE OF NEW-HAMPSHIRE, ROCKINGHAM, ss., G., May 10th, 1837. Personally appearing the abovenamed A. B. and M. B., acknowledged the foregoing instrument to be their voluntary act and deed.

Before me, S. A., *Justice Peace.*

The person who conveys by Deed is called the Grantor; he who receives the Deed is called the Grantee. Where the Deed is with Warrantee, the Grantor is also the Warrantor. The person who gives a Bond is the Obligor, and he who receives it is the Obligee. The person giving a Lease is the Lessor, the individual receiving it is the Lessee.

The difference between a Warrantee and Quitclaim Deed, is that in a Warrantee Deed the Grantor warrants and defends against all persons whomsoever; in a Quitclaim Deed, the Grantor warrants his own title merely, and against all persons claiming under him.

FORM OF A BOND.

Know all men by these Presents, that I [we] A. B., of C., in the County of R., and State of New-Hampshire, gentleman, [and C. D., of, &c.] am [are] held and firmly bound to E. F., of said C., yeoman, [and G. H., of, &c.] in the sum of one hundred dollars, to be paid to the said E. F. [and G. H., or either of them,] or his [their] certain attorney, executors, administrators or assigns, to which payment, well and truly to be made, I [we jointly and severally] bind myself [ourselves] my [our] heirs, executors and administrators firmly by these presents.

Sealed with my [our] seal [s], and dated the tenth day of May, A. D., 1837.

With a Condition to pay money.

The condition of this Obligation is such, that if the said A. B. [C. D., &c., or either of them] his [their] heirs, executors and administrators, do and shall well and truly pay or cause to be paid to the said E. F. [G. H., or either of them] his [their] executors, administrators or assigns, the sum of fifty dollars and interest, on or before the tenth day of May next; then this obligation to be void, otherwise in force.

[*Signed and sealed as the preceding forms.*]

GENERAL FORM OF AN AGREEMENT.

Articles of Agreement indented, made and concluded the tenth day of May, in the year of our Lord one thousand eight hundred and thirty-seven (or A. D., 1837,) between A. B., of C., in the county of M., yeoman, of the one part, and C. D., of said C., husbandman, on the other part.

The said A. B., for the consideration hereafter mentioned, doth hereby covenant and agree, that, &c.

And the said C. D. doth hereby covenant and agree, &c.

In testimony whereof they have hereunto interchangeably set their hands and seals, the day and year above *(or first above)* written.

[*Signed and sealed as the preceding forms.*]

FORM OF A WARRANT OF ATTORNEY,

TO CONFESS JUDGMENT.

To A. B., Esquire, of, &c., an attorney of —— court of ——, to be holden at ——, on the —— day of ——, or to any other attorney of said court.

THIS is to authorize you, or any of you, to appear for me, E. F., of, &c., in the said court, or any other subsequent term, at the suit of G. H., of, &c., and by *non sum informatus, nil dicit*, or otherwise, confess judgment against me unto him, the said G. H., in an action of debt for one hundred dollars and costs of suit; and for your or either of your so doing, this shall be your warrant.

In witness whereof, I have hereunto set my hand and seal, this tenth day of June.

BRIEF FORM OF A RELEASE.

May 20, 1837. I, A. B., do hereby release to E. E. all suits, promises, covenants and demands, which I have or can claim against him.

[*Signed, sealed and witnessed, as other instruments.*]

NOTE. By a release of all demands are barred all rights and titles to lands, warrants, debts. duties, actions, judgments and executions, and all contracts except those which are to be performed on a future contingency. By a release of all covenants and promises, are released all such covenants and contracts as are not released by the word demand, &c.

ERRATA.

PAGE 26, Questions 9 and 10, for 28 years, read 340.
" 34, Ques. 33, for 92, read 94.
" 51, Ques. 1, for $124,05, read $44,23.
" 86, Third paragraph, 3d line, for *dividend* read *divisor*.
" 101, Ques. 1, for *his share*, read *the ship*.
" 116, Ques. 11, for 7*s*., read 8*s*. Ques. 16, for $\frac{2}{3}$, read $\frac{1}{6}$.
" 120, First line of the Rule, for *on*, read *or*.
" 133, Third paragraph, 12th line, for $\frac{12}{16}$, read $\frac{16}{12}$.
" 136, Ques. 3, for 5×5, read 5×8.
" 140, Ques. 60, *Ans.*, for 10$\frac{3}{5}$ *inches*, read 2 ft. 10$\frac{3}{5}$ *in*.

ERRATA—CONCLUDED.

PAGE 53, Question 2, Answer for 31, read 3,1.
" 57, " 2, " for 365, read 3650.
" 68, " 54, " for 19135, read 17087.
" 91, " 8, " for $\frac{9}{14}$, read $\frac{3}{14}$.
" 94, " 6, " for 51, read 50.
" 96, " 8, for $\frac{27}{57}$, read $\frac{27}{54}$.
" 98, " 1, for $\frac{2}{4}$ read $\frac{3}{4}$.
" 103, " 2, Ans. for $\frac{3396}{16632}$, read $\frac{3696}{16632}$.
" 103, " 3, " for $3\frac{1}{16}$, read $2\frac{103}{120}$.
" 103, " 7, for $\frac{19}{26}$, read $\frac{19}{20}$.
" 104, " 7, Ans. for $\frac{43}{110}$, read $1\frac{7}{440}$.
" 110, " 28, erase (of $\frac{1}{14}$.)
" 130, " Answer, 3, for $600, read $6,00.
" 131, " 7, for $87\frac{1}{2}$, read 10 cts.
" 131, " 13, " for 470, read 270.
" 149, " 3, " for $6360,25, read 2975,258.
" 150, " 10, " for 19*s*. 10*d*. 3*qs*.,read 9*s*. 11*d*. 1*q*.+.
" 151, " 14, for $35\frac{1}{4}$, read $35\frac{1}{4}$*d*.
" 151, " 18,Ans., for $15\frac{1}{2}$, read 18 crowns.
" 155, " 8, " for $\$3,93\frac{1}{3}$, read $3,633\frac{1}{3}$.
" 156, " 12, " for ,628, read ,728.
" 157, " 23, " for ,913, read ,924+.
" 158, " 31, for amount, read interest.
" 158, " 36, Answer, for $418,16, read 1418, 75.
" 174, " 16, " for £12 0*s*.&c.,read£9 12*s*. 3*d*. 2*qs*.+.
for $19\frac{3}{13}$, read $15\frac{3}{10}$+ pr. cent.
" 174, " 17, " for $6,669, read $6,695.
" 177, " 10, " for $500,25, read 2200,25.
" 178, " 2, " for $1245,175, read $1250,185.
" 178, " 7, " for ,984, read ,913.
" 179, " 19, " for *L*47, &c. read *L*37 1*s*. 7*d*. 1*q*.
" 179, " 20, " for $875,674, read $975,345.
" 180, " 6, " for 5 months, read 3 months.
" 186, " 5, for B's. loss $89, read $86.
" 189, for the cube root of 359-14, read 357-14.
" 192, Ques. 9, Ans., for ,7177, read ,7207.
" 193, " 18, " for ,223+, read ,707+.
" 195, " 37, " for 32+, read 28.
" 199, " 1, " for 64, read 4.
" 200, " 3, " for 243, read 343.
" 212, 10, " for 198, read 196.
" 223, " 11, 3d line, for $11, read $10.
" 226, " 3, Ans. for 156ft.. read 231ft.
" 232, " for 72ft. read 78.
" 237, " 28, Ans. for $11,914, read 12,014.
" 238, " 36, " for $197,05, read 198,01.
" 238, " 43, " for *L*20, read *L*22.
" 293, " 53, for 474512, read 474552.
" 293, " 56, Ans. for $10\frac{2}{3}$, read 6 hours.
" 245, " 113, " for 20106,176, read 201062,4.